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Optimality of bailout lending when asymmetric information increases sovereign yields

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Abstract

Is the sovereign debt market information-sensitive to the true borrowing amount? If yes, does asymmetric information affect welfare? What mechanism can improve the welfare of transparent governments? To address these questions, this paper builds a sovereign debt model in which the government can be transparent or opaque. The model predicts that asymmetric information about the government's type decreases the welfare of a transparent government, by either inducing a transfer of welfare to an opaque government through the market price or by leading to market breakdown. Building on this model, I develop a mechanism of inspection and penalties on cheating governments and conclude that bailout lending when market yields rise as a result of this information friction does not necessarily improve the welfare of transparent governments compared to market lending, if the market still provides lending.

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1. Introduction

In October 2009, the newly elected government in Greece announced a revision of the estimate for its 2009 deficit from 6% - 8% to 12,5%, saying that estimates provided by the previous government had been significantly understated¹. It also revised Greece's 2008 deficit from 5% to 7,7%². Greek sovereign yields immediately soared.

In January 2010, a report published by the European Commission³ said that Greece's figures were "so unreliable that its budget deficit and public debt could be higher than the numbers claimed by the Greek government in October". According to the news⁴, the report said that "a substantial number of unanswered questions and pending issues still remain in some key areas (...) and it cannot be excluded that this will lead to further revisions of Greek government deficit and debt data, particularly for 2008 but possibly also for previous years". In April 2010, the estimate was again revised upwards and the final estimate is 15,2%⁵. In May 2010, Greece requested an official bailout from the IMF and the EU⁶.

Although the Commission said that EU fiscal data were generally of high quality and that Greece was an isolated case, it also said that the EU lacked audit powers and so "relied heavily on the goodwill and integrity of member-states to supply accurate data". Facts are that in January 2010 sovereign yields in Portugal also sky-rocketed, without any policy announcement at the national or European level about Portugal, and in May 2011 Portugal requested an official bailout from the IMF and the EU⁷.

¹ Financial Times, October 20, 2009 "Greece vows action to cut budget deficit"

² Financial Times, January 12, 2010 "Greece condemned for falsifying data"

³ Brussels, 8.1.2010 COM(2010) 1 final "Report on Greek Government deficit and debt statistics"

⁴ Financial Times, January 12, 2010 "Greece condemned for falsifying data"

⁵ Source: Eurostat

⁶ IMF Press Release No. 10/176, May 2, 2010

⁷ IMF Press Release No. 11/190, May 20, 2011

Figure 1: Greece's sovereign yields (%)⁸

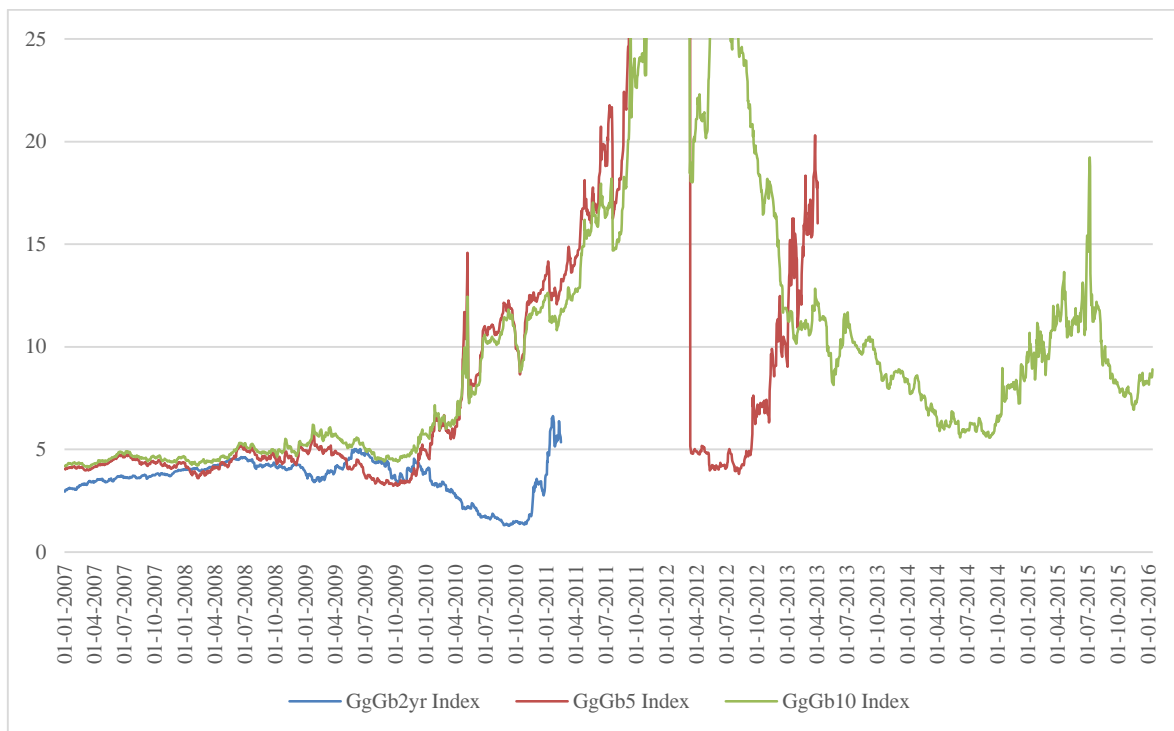
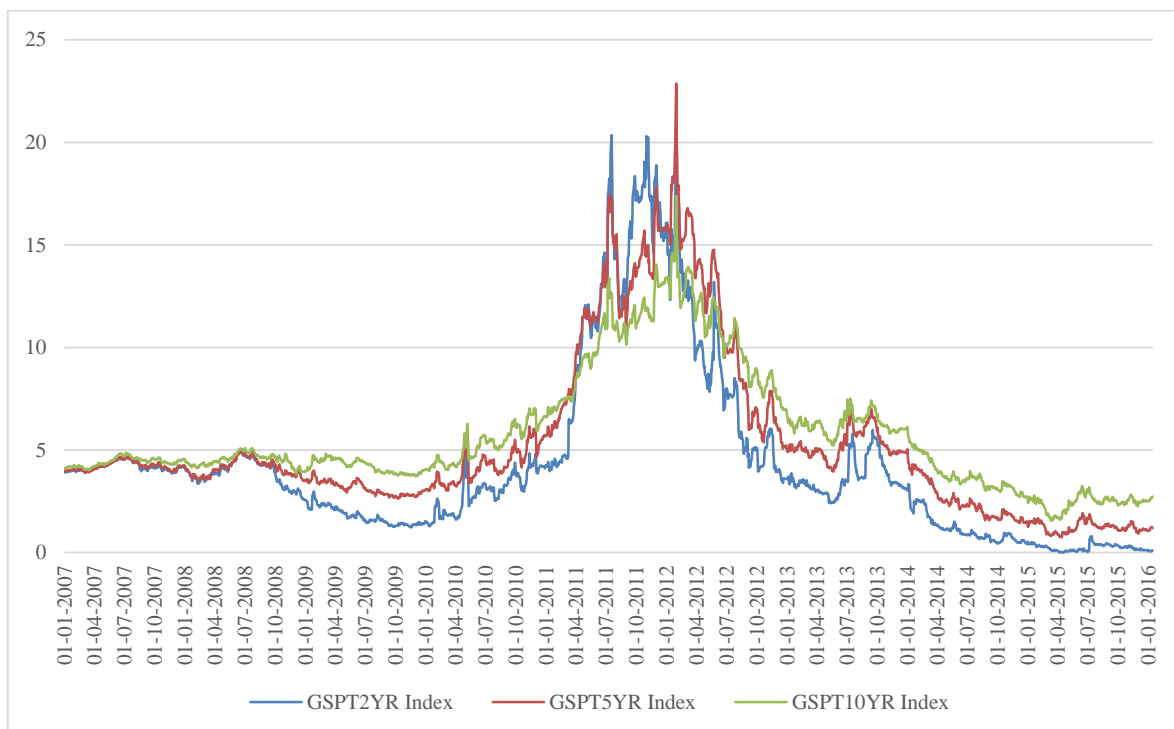


Figure 2: Portugal's sovereign yields (%)⁹



⁸ Source: Bloomberg

⁹ Source: Bloomberg

The recent sovereign debt crisis in Europe revived the role that information might have on the sovereign debt market. Kletzer (1984) showed that lack of observability of the true amount of borrowing by a government leads to a loss of welfare, because lenders treat the government as a price taker rather than as a price maker. The loss of the first-mover advantage implies that the equilibrium of the market under lack of observability is weakly dominated by the equilibrium of the market under observability. Kletzer's empirical motivation was the experience of the less-developed countries, during the late 1970s. The empirical motivation of this paper is the substantial rise of Portugal's sovereign debt yields in 2010, shortly after Greece revealed that it had misreported its true borrowing levels. Although investors did not know whether Portugal also misreported its numbers or not, one interpretation is that the Greek revelation showed that it *was possible* to hide debt even within the EU.

This paper therefore aims at building a formal structure to study the effect of *not knowing* whether a government can hide debt or not. Building upon the model, the second aim of this paper is to investigate the effect of allowing for a mechanism of inspection with penalties for cheating, which can be interpreted as bailout lending or as the introduction of penalty clauses on bond contracts enforceable by a supranational authority, on the welfare of a well-behaved government. Given the lack of such a clear supranational authority in reality, this paper can be best seen as a model of bailout lending under asymmetric information.

The structure of this thesis is the following: chapter I studies the information-sensitivity of the market by building a sovereign debt model with one government type that is transparent and one type that is opaque, where type is private information to the government. It also studies the effect of asymmetric information on welfare. Chapter II develops a mechanism of costly inspection of the government's true amount of borrowing and infliction of penalties on a government found cheating. Chapter III analyzes the extent to which the introduction of this

mechanism increases the welfare of each government type. The paper ends with a summary of the conclusions.

2. Relation with the literature

The central theme of the sovereign debt literature is the risk of repudiation. The breakthrough was made by Eaton and Gersovitz (1981), where a theory about sovereign default was first developed. The theory says that sovereign lending exists in equilibrium to the extent that a government sustains a reputation as a good debtor. It is the fear of being excluded from the market following a default that keeps sovereign debtors repaying. In that sense, the more a country values inter-temporal consumption smoothing, for example, the higher the amounts of debt that it will be served by the market, since investors know about the government's incentives to repay. This is called the reputational approach to sovereign debt and has been followed by several authors throughout time (Kletzer (1984), Grossman and Van Huyck (1988), Cole and Kehoe (1998 and 2000), Wright (2002) and Arellano (2008), for example).

A parallel theory about sovereign default was developed by Bulow and Rogoff (1989), defending that lending to sovereigns is supported by the extent to which lenders have legal rights to impose sanctions on a defaulting government, such as impeding a country's trade or seizing its financial assets abroad. This approach – the direct sanctions approach – differs substantially from the reputational one, since, precisely, the reputational approach lies on the assumption that sovereign lending does not, by the nature of a sovereign as an immune debtor, rely on collateral. Both approaches explain the existence of debt ceilings on governments debt (that is, threshold levels of debt above which lenders are not willing to lend, even though a sovereign might be willing to pay a higher interest rate) and both model the price schedule of loans as a decreasing price schedule on the amount borrowed (the higher the amount of the loan, the higher the relative incentive to repudiate on it, according to the reputational approach, and the lower the collateral per unit of debt, according to the direct sanctions approach). Both

these features induce a decrease on the welfare of the sovereign, when compared to perfectly enforceable debt.

Formally, time consistency is a fundamental requirement when formalizing a sovereign debt market equilibrium, as sovereign debtors decide to pay or to default at each period in time weighing the loss of repaying against the loss of defaulting, as opposed to following a previously defined repayment plan. Accordingly, the planning/Ramsey formulation should not be used to formalize the equilibrium, since the latter does not require time consistency, when the agents can re-optimize within periods (that is, subgame-perfection in a sequential game).

As a result of imperfect enforcement and sequential decision-making, the literature explains limited risk-sharing in the sovereign debt market between risk-averse sovereign debtors and risk neutral lenders (for example, Grossman and Van Huyck (1988)). Sovereign debt is widely modeled as state-non contingent bonds, because state-contingent debt is not sustainable in equilibrium when the debtor can repudiate on the contingencies established in the contract when it benefits from doing so.

The introduction of the possibility to reschedule (or, more broadly, to renegotiate) sovereign debt enriched the descriptive power of sovereign debt models. Allowing for debt renegotiation impacts the debt ceilings imposed on sovereigns, since lenders can receive partial repayments (a model of partial default with a simple modeling of bargaining powers can be found in Sachs and Cohen (1982), while models of partial default with more developed bargaining processes can be found in Fernandez and Rosenthal (1990) and Bulow and Rogoff (1989)).

The problem of multiplicity of equilibria has been studied in this market. The ways in which multiplicity may arise are studied, for example, by Calvo (1988), Cole and Kehoe (2000) and Ayres, Navarro, Nicolini and Teles (2015).

Quantitative models have also been built with country-specific calibrations, allowing for positive explanations of empirically observed events in sovereign debt markets (for example, Aguiar and Gopinath (2006) and Arellano (2008)).

Although the sovereign debt market literature has developed continuously throughout time, the topic of information frictions remains relatively understudied. Kletzer (1984) was the first to bring about the role of the information structure in the sovereign debt market. Kletzer's main insight is that lack of observability of the true amount of borrowing by a government leads to a loss of welfare, because lenders treat the government as a price taker rather than as a price maker. The loss of the first-mover advantage is abstracted from in the majority of sovereign debt models, since they implicitly assume that investors are able to know the amount of debt that they are pricing. The order in which players move that this introduces can be related to Ayres et al. (2015) and Lorenzoni and Werning (2013), since both those papers also change the usually implicit assumptions regarding the timing of the borrowing relationship. Lorenzoni and Werning explain it as a lack of commitment to previous borrowing announcements, since the borrowing needs of the government are only realized after lenders have fixed prices. In this model, I explain timing differences with asymmetric information. My model and those other two share in common the fact that all departure from the standard sovereign debt assumption of the first-mover timing, which leads to important qualitative changes in the results.

Atkeson (1991) studies asymmetric information regarding the use of borrowed funds. He assumes that lenders are unable to observe whether the government uses loans for consumption or investment and assumes that lenders cannot perfectly inspect the use of borrowed funds by the government, for which reason the latter has incentives to overuse the funds for consumption. There, loan repayment is more attractive for higher levels of output and output increases with investment. The moral hazard potential to overuse borrowed funds for consumption makes the optimal contract demand high repayments when the realization of income is so low that it

indicates less than optimal investment. This information asymmetry and its consequences on the optimal debt contract are meant to provide an explanation for why state contingent contracts provide only partial insurance to risk-averse governments in face of adverse output shocks.

The asymmetric information price schedule of my model accounts for the risk of debt dilution. Debt dilution risk and the resulting haircut on the price decrease welfare, by inducing adverse selection and increasing the cost of debt. Therefore, the search for a remedy to overcome the problem of debt dilution proves important. The topic of debt dilution and the study of remedies to overcome that distortion have been present in the literature. One proposed solution is the introduction of seniority clauses (see for example Chatterjee and Eyigungor (2015)). However, remedies such as seniority clauses would not overcome the debt dilution problem if it arises due to asymmetries of information. The introduction of costly inspection is adequate to deal with unobserved action (see Townsend (1979)). Additionally, in my model, the mechanism designed does not suffer from the existence problem present in Rothschild and Stiglitz (1976), because the amount of the penalty is exogenous to the model.

3. Chapter I: information-sensitivity of the market

3.1 The baseline model

3.1.1 Fundamentals (time horizon, agents, endowments and payoffs)

There are two periods, denoted $t \in \{0,1\}$. At $t = 0$, the government receives an exogenous endowment of $y_0 = 0$, and has a legacy debt $c > 0$. At $t = 1$, the government receives an exogenous endowment y_1 , which is randomly drawn from a uniform distribution, whose support is $[y_0, \bar{y}]$, where \bar{y} takes the value of 1. The government is risk neutral, does not discount the future and maximizes the following linear lifetime payoff over consumption:

$$c_0 + c_1$$

A set of small investors has funds which they can lend to the government, in the form of state-non contingent discount bonds B at a price q with one period maturity. Since lenders are small,

no individual lender can be a single lender to the government. Lenders are risk neutral, do not discount the future and are perfectly competitive. π denotes the profit from investing in a sovereign bond. The expected profit at the time of the investment is:

$$E(\pi) = -qB + B * \text{repayment probability} + 0 * \text{default probability}$$

In equilibrium, the bond price q must satisfy:

$$E(\pi) = 0$$

The government can borrow B from lenders at $t = 0$. If $qB < c$, the government defaults at $t = 0$, since $c > y_0$. If the government defaults at $t = 0$, it does not move on to period 1. If it repays, it moves on and receives y_1 at $t = 1$. At $t = 1$, the government can either repay its loan and receive $y_1 - B$ or repudiate on its loan and receive y_0 , the lower bound of the distribution of y_1 (this default cost follows Ayres et al. (2015)).

3.1.2 Information structure

There are two government types: type 1 can only borrow B and type 2 can borrow B and H , with H taking values in $[0, K]$. Within this environment, I analyze two information structures: first, a structure of symmetric information, in which the government's type is public information; second, a structure of asymmetric information, where the government's type is private information to the government. A type 1 government can be seen as a transparent government, which cannot deviate from the borrowing announcement made to the public. A type 2 can be seen as a government who can end up borrowing more than the amount announced by the time lenders fix the bond price. In the model, I say that a type 2 can *hide* borrowing.

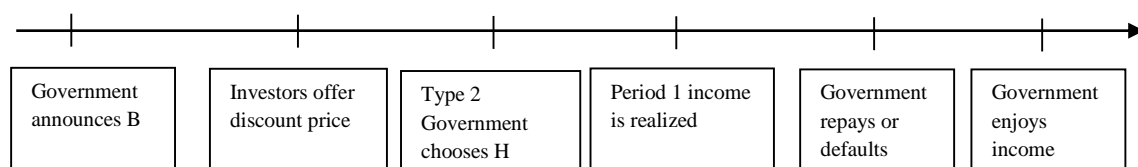
3.1.3 Motivation to borrow and gains from trade

$y_0 < c$, which implies that the government needs to borrow in order to avoid default at $t = 0$. If the government defaults at $t = 0$, its payoff is 0. If the government repays, it moves on to period 1 and its period 1 payoff is bounded below by 0, since the government can repudiate on its loan and the default cost is a payoff of 0. Hence, borrowing at $t = 0$ weakly dominates

autarky. Lenders make zero expected profits in a lending equilibrium, so they have a payoff of 0 either by participating in the market or by not participating. So aggregate surplus is weakly higher in a lending equilibrium than in an autarky one. The gains from trade are the transfer of wealth that the government can make between periods through trading bonds.

3.1.4 Timing

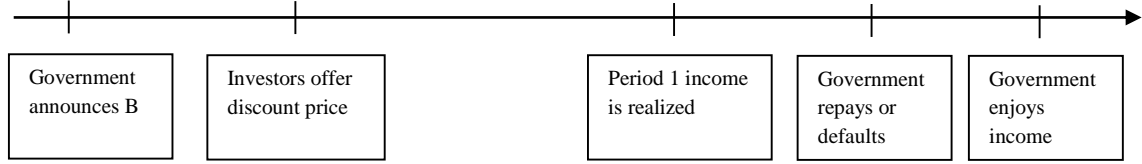
Formally, the government's type translates into a specific timing of the loan relationship, since private information regarding the government's type introduces an information friction which is taken into account by lenders when pricing bonds. If the government is of type 1, it moves first by announcing B and lenders move second, by offering q . If the government is of type 2, the government moves first by announcing B , then lenders offer q , but then the government moves again by choosing H . Both B and H affect the repayment probability of B , so lenders take into account what they anticipate the choice of H to be when choosing q . In that sense, the government behaves as a price-taker in H . The timing of the game is the following: at $t = 0$, both types announce B . Then investors decide q through take-it-or-leave-it offers. Then, a type 2 has another move, which is the choice of H . At $t = 1$, income y_1 is exogenously realized and a type 1 chooses to repay or default on B and a type 2 chooses to repay or default on $B + H$. After a default, both types enjoy the autarky payoff while repayment yields y_1 net of paid funds. This is summarized in the timeline below:



3.2 Equilibrium

3.2.1 Equilibrium under symmetric information and a type 1 government

When lenders know they face a type 1 government, the timing of the model becomes the following:



A. Decision problem of the government

At $t = 1$, the payoff from defaulting is:

$$y_0$$

The payoff from repaying is:

$$y_1 - B$$

The payoff-maximizing government therefore repays at $t = 1$ if and only if:

$$y_1 > y_0 + B$$

At $t = 0$, the government chooses the amount of debt to borrow (B) in such a way as to maximize its lifetime payoff, knowing that it will optimally choose to repay or default at $t = 1$.

So the government chooses B to solve the following problem:

$$\max_B y_0 - c + qB + \int_0^B (0) dF(y_1) + \int_B^1 (y_1 - B) dF(y_1)$$

The initial part of the objective function is the government's payoff at $t = 0$, which is its endowment net of bills plus the proceeds from borrowing. The second part of the objective function is the government's expected payoff at $t = 1$, which is either its endowment net of debt repayment or the autarky income, taking into account the optimal repayment choice. The repayment decision is probabilistic at $t = 0$, since the distribution of y_1 is public information. The decision to repay follows the spirit of the sovereign debt literature in that it is a strategic decision: a government repudiates on its debt unless its payoff is higher if it repays. However, this is not apparent in this model, since the default cost is modeled to be the lower bound of the distribution of period 1's income. Hence, the choice to repay or default becomes mechanical and determined by ability-to-pay, since the default cost is as strong as an income of 0.

B. Equilibrium bond price

Since each creditor is small, under perfect competition the requirement for zero expected profits implies that they all offer the same price. Since the risk-free rate in this model is 0, the expected return of lending in this economy must be 0.

$$E(\pi) = 0$$

Since bonds are discount bonds, the equilibrium bond price is pinned down by the relationship:

$$q = 1 - B$$

The bond price is the repayment probability of the loan, since, for each unit lent, lenders receive one unit in case of repayment and zero in case of default. This bond price schedule is the supply of loan contracts, since it defines a set of pairs of loan amounts and prices that achieve zero expected profits, given optimal default behavior by the government.

Moreover, lenders will not lend amounts of debt for which the probability of repayment is 0, so, in equilibrium, it must be the case that:

$$B \leq 1$$

Imperfect enforcement is the reason behind a decreasing bond price schedule and credit ceilings on sovereigns. As the loan increases, so does default probability, which decreases the price.

C. Equilibrium lending

Equilibrium is defined as: prices clear markets given available information and quantities are optimal. An equilibrium is then a pair (q, B) which solves:

$$\max_B -c + qB + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - B) dF(y_1)$$

$$s. t.$$

$$q = 1 - B$$

$$qB \geq c$$

$$B \leq 1$$

The first constraint is the zero profit condition. The second constraint is the government's budget constraint. The third constraint is the debt ceiling. The equilibrium borrowing amount can be shown to be:

$$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$$

This equilibrium amount is the minimum amount necessary to pay back bills c and avoid default at $t = 0$. The intuition is the following: a decreasing loan price schedule implies that the marginal benefit of debt is lower than its marginal cost, for any debt size.

$$\text{marginal benefit of } B = 1 - 2B$$

$$\text{marginal cost of } B = 1 - B$$

While the marginal benefit of debt is its repayment probability, since that is what lenders receive per unit of debt in expected terms, the marginal cost of debt is the repayment probability of the loan in case of repayment and the default cost in case of default. Since the default cost is not zero (the government loses its period 1's income), the expected cost of the loan is higher than its repayment probability. This results from the fact that, although sovereigns can repudiate on their loans, they do suffer a cost for defaulting, which is not received by the creditors.

Since the government's payoff is decreasing in B , there must be an upper bound on c for which it is individually rational to borrow. Since the government's payoff at $t = 1$ is bounded below by 0, borrowing is individually rational insofar as c can be raised at the prevailing price schedule. Therefore, borrowing is individually rational for $c < qB$. With B optimally chosen, this yields the upper bound $c < \frac{1}{4}$. These findings are summarized in Lemma 1.

Lemma 1: *With symmetric information, lending exists in equilibrium for $c < \frac{1}{4}$ for a type 1 government.*

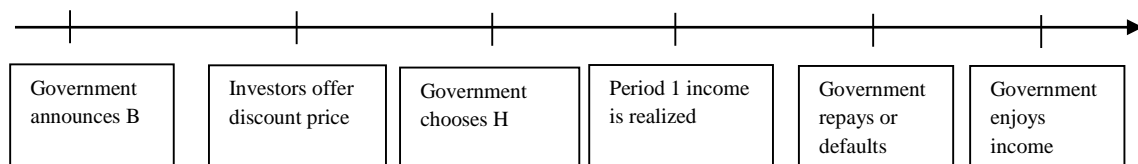
Table 1 describes the market equilibrium as a function of c :

Table 1: Summary of market equilibrium under symmetric information and type 1

Information structure	Symmetric (Type 1 Government)	
c (funding needs)	$c \leq \frac{1}{4}$	$c > \frac{1}{4}$
Equilibrium Borrowing	$B = \frac{1}{2} - \frac{1}{2}\sqrt{1-4c}$	0
Equilibrium price schedule	$q = \frac{1}{2} + \frac{1}{2}\sqrt{1-4c}$	$q = 0$
Expected payoff Government	$\frac{1}{4} + \frac{1}{4}\sqrt{1-4c} - \frac{c}{2}$	0
Lenders' expected profit	0	0
Default probability	$\frac{1}{2} - \frac{1}{2}\sqrt{1-4c}$	0

3.2.2 Equilibrium under symmetric information and a type 2 government

When lenders know they face a type 2 government, the timing becomes:



A. Decision problem of the government

The problem is solved by backward induction, as in the case of a type 1 government, except that there is one additional layer of decision making, which is the government's choice of H .

At $t = 1$, the payoff from defaulting is:

$$y_0$$

The payoff from not defaulting is:

$$y_1 - B - H$$

Therefore, a payoff maximizing government repays at $t = 1$ if and only if:

$$y_1 > y_0 + B + H$$

At $t = 0$, after a type 2 has announced B and lenders have offered q , a type 2 chooses H to maximize its payoff. The choice of H therefore solves the following problem:

$$\max_H y_0 - c + q * (B + H) + \int_0^{B+H} 0 dF(y_1) + \int_{B+H}^1 (y_1 - (B + H)) dF(y_1)$$

The solution is that the hiding choice of a type 2 government is a corner solution: either 0 or K . Since lenders do not observe H , the price of B is not a function of H . In order for it to be incentive compatible for a type 2 to hide a positive amount of H , the price has to be sufficiently attractive. A sufficiently attractive price is a price that is sufficiently higher than the repayment probability of $B + H$. Only in that case is the marginal benefit of loans higher than its marginal cost and it is optimal to choose a borrowing amount larger than the necessary to avoid default in period 0. Otherwise, the increase in consumption at $t = 0$ attained through hiding is not enough to compensate for the cost in period 1 of that same hiding (an increase in repayment or a higher probability of default) and the government's payoff actually decreases by choosing a positive amount of H . Formally, lenders choose to hide H if and only if:

$$\begin{aligned} \max_B y_0 - c + q * (B + H) + \int_0^{B+H} 0 dF(y_1) + \int_{B+H}^1 (y_1 - (B + H)) dF(y_1) \\ > \max_B y_0 - c + q * (B) + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - (B)) dF(y_1) \end{aligned}$$

This yields a discontinuous demand function for H : either 0 or K . In my model, since the government is risk neutral and the productivity of loans is their discount price, which, once fixed, does not decrease with loans, if it is profitable to borrow one additional unit once the price is fixed, it is profitable to borrow all possible additional units. Although a higher default probability decreases the government's future expected consumption, the inter-temporal allocation of consumption is not a concern under risk neutrality.

B. Equilibrium bond price

With perfect competition, the price schedule is pinned down by the relation:

$$E(\pi) = 0$$

Since lenders cannot observe H , the supply of loans schedule is of the form:

$$q = 1 - B - x$$

Where x is the conjectured level of hiding.

Fact 1: *With symmetric information about the government's type, investors only lend to a type 2 government on its demand curve.*

A demand curve amounts to an incentive compatibility constraint. In this model, since the choice of H is either 0 or K and since if lenders price $H = 0$ the price is sufficiently high for a type 2 government to deviate to $H = K$, a necessary condition for a lending equilibrium is that lenders price the default risk of K . So the equilibrium bond price schedule is:

$$q = 1 - B - K$$

Moreover, lenders will not lend amounts of debt for which the probability of repayment is 0, so in equilibrium it has to be the case that:

$$B + K \leq 1$$

C. Equilibrium lending

The equilibrium price schedule is such that there is no loan amount (including hidden borrowing) whose price can be higher than its repayment probability, so there is no opportunity for profitable hiding. Since hiding is unprofitable, the optimal choice of total debt is the minimum necessary to avoid default at $t = 0$. So the equilibrium B solves:

$$c = (1 - B - K) * (B + K)$$

Which is:

$$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c} - K$$

And the choice of H is:

$$H = K$$

Since hiding is unprofitable, the government will borrow as least as possible. That is achieved by decreasing B by K . It must be IR for a type 2 government to participate in the market. Since borrowing is IR as long as it can raise its funding needs in the market so as to avoid default at $t = 0$, the binding constraint on c is the price schedule. Since B is decreased by K , the price schedule ends up being equal to that of a type 1, which yields the same upper bound on c .

Lemma 2: *With symmetric information about the government's type and about the value of K , lending exists in equilibrium for $c < \frac{1}{4}$ for a type 2 government.*

Under asymmetric information about total borrowing, the market is only in equilibrium if there is a price for which the demand for loans of the government yields zero expected profits. Here, however, an interest rate equilibrium is exactly the same as the equilibrium when the government is a first mover (the amount of debt is pinned down by c , regardless of whether it consists only of B or of $B + K$), because, when the price yields zero profits to lenders, hiding is unprofitable. So symmetric information does not dominate asymmetric information.

Proposition 1: *With symmetric information about the government's type and about the value of K , welfare is invariant to the ability to hide debt.*

Table 2 describes the market equilibrium as a function of c :

Table 2 : Market equilibrium under symmetric information and type 2

Information structure: Symmetric (type 2 government)		
c (funding needs)	$c \leq \frac{1}{4}$	$c > \frac{1}{4}$
Equilibrium Borrowing	$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c} - K$	0
Equilibrium H	K	0
Equilibrium Price Schedule	$q = 1 - B - K$	0

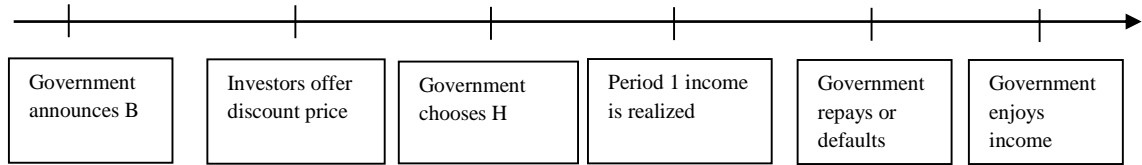
Expected payoff Government	$\frac{1}{4} + \frac{1}{4}\sqrt{1-4c} - \frac{c}{2}$	0
Lenders' expected profit	0	0
Default probability	$\frac{1}{2} - \frac{1}{2}\sqrt{1-4c}$	0

3.2.3 Equilibrium under asymmetric information about type

In this section, I introduce the information friction, in the form of private type, that is, lenders do not know which government type they face when offering prices. This is meant to formalize the uncertainty regarding the level of transparency of countries' borrowing data, after the Greek revelations. By solving the problem under asymmetric information, it is possible to draw conclusions about the information sensitivity of this market and the resulting impact on welfare.

Definition 1: Asymmetric information about type consists of lenders believing that the government is of type 1 with probability $1 - P$ and of type 2 with probability P .

The timing of the game is now the following:



A. Decision problem of the government

At $t = 0$, a type 1 government chooses B to maximize its lifetime expected payoff:

$$\max_B y_0 - c + qB + \int_0^B (0) dF(y_1) + \int_B^1 (y_1 - B) dF(y_1)$$

At $t = 0$, taking B as given, a type 2 chooses $H = K$ if and only if (and $H = 0$ otherwise):

$$\begin{aligned} \max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0 dF(y_1) + \int_{B+K}^1 (y_1 - (B + K)) dF(y_1) \\ > \max_B y_0 - c + q * (B) + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - (B)) dF(y_1) \end{aligned}$$

And chooses B to maximize its lifetime expected payoff, taking H as given:

$$\max_B y_0 - c + q * (B + H) + \int_0^{B+H} 0 dF(y_1) + \int_{B+H}^1 (y_1 - (B + H)) dF(y_1)$$

B. Equilibrium bond price

Since a type 2's choice of H is optimally binary (either 0 or K) and since pricing $H = 0$ makes it optimal for a type 2 to choose $H = K$, which is cannot be an equilibrium (lenders would be making a loss), a necessary condition for the equilibrium price is that it assumes that a type 2 chooses $H=K$. So, in equilibrium, lenders' payoff must be:

$$E(\pi) = -qB + P(B(1 - B - K)) + (1 - P)(B(1 - B))$$

The risk of default is the risk of default of a type 1 with probability $(1-P)$ and that of a type 2, with probability P . Equating expected profits to zero yields the following price schedule:

$$q = 1 - B - PK$$

Intuitively, if lenders are uncertain about the government's ability to hide debt, they are willing to incur in a loss in case the government can indeed hide debt, as long as they incur in a profit, in case the government cannot hide debt. The price is unfairly low to a well-behaved government and unfairly high to a bad government. If a new lender lowers the discount price, the government will not prefer her loans, since the revenues per unit of debt are lower. If a new lender increases the discount price, she will make an expected loss, since a type 2 will continue to borrow K . So this is a Nash-equilibrium.

C. Equilibrium lending

This price schedule can implement a pooling equilibrium, in which a type 2 does not reveal its type (hence the equilibrium is consistent with lenders belief P) and hides K (hence the price

schedule achieves zero expected profits) if and only if it is IC for a type 2. For a type 2, it is incentive-compatible to hide K if the price is sufficiently high. Hence, a type 2 chooses $H = K$ and not $H = 0$ if and only if:

$$\begin{aligned} \max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0 dF(y_1) + \int_{B+K}^1 (y_1 - (B + K)) dF(y_1) \\ > \max_B y_0 - c + q * (B) + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - (B)) dF(y_1) \end{aligned}$$

At a price schedule of

$$q(B) = 1 - B - PK$$

It can be shown that this only happens if $P < \frac{1}{2}$. In that case, hiding is profitable, for which reason a type 2 will mimic the choice of B of a type 1 and not reveal information about its type. Therefore, the equilibrium B for both types solves:

$$\max_B -c + qB + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - B) dF(y_1)$$

s. t.

$$qB \geq c$$

$$q(B) = 1 - B - PK$$

$$B < 1$$

Which is

$$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c}$$

In order for this pooling equilibrium to hold, it must be IR for a type 1. The highest c for which it is IR for a type 1 is defined by the price schedule, as usual. It is the maximum of the function qB at the price schedule $q(B) = 1 - B - PK$, which yields $c < \frac{1}{4} - PK \left(\frac{2-PK}{4} \right)$.

Hence, for $c < \frac{1}{4} - PK \left(\frac{2-PK}{4} \right)$, the market is in a pooling equilibrium, in which both types borrow the same amount of B and a type 2 borrows K . The well-behaved government ends up

paying a rent (through the market price) to a type 2 in order to be able to participate in the market.

For $c > \frac{1}{4} - PK \left(\frac{2-PK}{4} \right)$, this equilibrium is not IR for a type 1, since the price schedule becomes negative, but it might be IR for a type 2, since it can make the price of B positive at the prevailing price schedule, by decreasing the demand for B and borrow K . However, it will signal its type and lenders would adapt the price schedule to a type 2. As a result, when $c > \frac{1}{4} - PK \left(\frac{2-PK}{4} \right)$, lenders offer $q(B) = 1 - B - K$, there is market breakdown for a type 1 government and a type 2 chooses B to solve the same problem as with symmetric information:

$$\max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0 dF(y_1) + \int_{B+K}^1 (y_1 - (B + K)) dF(y_1)$$

s. t.

$$q(B + K) \geq c$$

$$q(B) = 1 - B - K$$

$$B + K < 1$$

Whose solution is:

$$B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4c} - K$$

$$H = K$$

The only lending equilibrium is then a separating equilibrium in which there is adverse selection, since only a type 2 participates in the market. A type 2 will participate up to $c < \frac{1}{4}$, which is the maximum amount of funds that can be raised in an equilibrium with symmetric information and a type 2.

If $P > \frac{1}{2}$, this asymmetric information price schedule does not make hiding IC (the price is not sufficiently higher than the repayment probability of the loan to make borrowing payoff-increasing). In this case, there is no zero expected profits price that can induce a bad type into

choosing the behavior it should choose for that price, except for a low enough price which makes it non-IR for a type 1 to borrow.

So the only possible lending equilibrium is one in which lenders offer a price schedule that makes it IR to borrow only for a type 2. Hence, there is a separating equilibrium, in which there is adverse selection, since only a type 2 participates in the market. It is only possible to separate types if $c > \frac{1}{4} - \frac{1}{2}K(1 - \frac{K}{2})$, since only then is it not IR for a type 2 to borrow at a type 2 price schedule ($q = 1 - B - K$). In this case, a type 2 will borrow B and K as in a symmetric equilibrium and will participate up to $c < \frac{1}{4}$, which is the maximum amount of funds that can be raised in an equilibrium with symmetric information and a type 2. These findings are summarized in Lemma 3.

Lemma 3: *With asymmetric information about the government's type, lending exists in equilibrium for $c < \frac{1}{4} - PK\left(\frac{2-PK}{4}\right)$ for a type 1 and for $c < \frac{1}{4}$ for a type 2, if $P < \frac{1}{2}$. If $P > \frac{1}{2}$, the market is in a zero lending equilibrium for any c for a type 1 and lending exists for $c > \frac{1}{4} - \frac{1}{2}K(1 - \frac{K}{2})$ for a type 2.*

Table 3 describes the market equilibrium under asymmetric information, as a function of c :

Table 3: Market equilibrium under asymmetric about type

Information structure	Asymmetric (Hidden Type)		
Prior beliefs	$P \leq \frac{1}{2}$		$P > \frac{1}{2}$
c (funding needs)	$c \leq \frac{1}{4} - \frac{PK(2 - PK)}{4}$	$\frac{1}{4} - \frac{PK(2 - PK)}{4} < c \leq \frac{1}{4}$	$c > \frac{1}{4}$

Equilibrium Borrowing	$Type\ 1 = Type\ 2: B$ $= \frac{1}{2}(1 - PK)$ $-\frac{1}{2}\sqrt{(1 - PK)^2 - 4c}$	$Type\ 1: B = 0$ $Type\ 2: B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$ $- K$	$Type\ 1$ $= Type\ 2: B$ $= 0$
Equilibrium Hidden borrowing	$Type\ 1: H = 0$ $Type\ 2: H = K$	$Type\ 1: H = 0$ $Type\ 1: H = K$	0
Equilibrium price schedule	$q = 1 - B - PK$	$q = 1 - B - K$	$q = 0$
Expected payoff Government	$Type1: \frac{1}{4}$ $+ \frac{1}{4}(1 + PK)\sqrt{(1 - PK)^2 - 4c}$ $-\frac{c}{2} + \frac{P^2K^2}{4}$ $Type2: \frac{1}{4}$ $+ \frac{1}{4}(1 + PK)\sqrt{(1 - PK)^2 - 4c}$ $-\frac{c}{2} + \frac{P^2K^2}{4} + K^2(\frac{1}{2} - P)$	$Type1: 0$ $Type\ 2: \frac{1}{4} + \frac{1}{4}\sqrt{1 - 4c} - \frac{c}{2}$	0
Average Expected payoff Government	$\frac{1}{4} + \frac{1}{4}(1 + PK)\sqrt{(1 - PK)^2 - 4c}$ $-\frac{c}{2} + \frac{P^2K^2}{4}$ $+ PK^2(\frac{1}{2} - P)$	$P\left(\frac{1}{4} + \frac{1}{4}\sqrt{1 - 4c} - \frac{c}{2}\right)$	0
Lenders' expected profit	0	0	0

3.3 Welfare

I use as measure of welfare the highest level of financing needs at which a government can be served in equilibrium. The higher such an upper bound, the higher the welfare provided by the equilibrium.

The government's welfare increases with the opportunity to borrow in the market, because without borrowing the government defaults on its bills and cannot move on to the next period. In case the government defaults, it achieves a payoff of zero and does not have access to next period's income, so his lifetime payoff is 0. On the contrary, if the government pays its bills, it can enjoy next period's income. Although the government carries a debt burden into the next period, enforcement of loans is imperfect, which safeguards that the worst scenario for the government next period is to suffer the default cost of repudiating on its debt. Since default cost next period equals default cost this period (a payoff of 0), borrowing weakly dominates autarky. Lenders make zero expected profits in equilibrium independently of providing lending or not, so aggregate welfare in this economy is the welfare of the government.

When there is symmetric information about the government's type, lending exists up to the point where imperfect enforcement allows it to exist. The ability to hide borrowing does not distort welfare if the government who can hide borrowing has a linear payoff function in consumption. In turn, asymmetric information about the government's type distorts welfare. Overall, asymmetric information makes welfare incomparable when $P < \frac{1}{2}$ (either higher or lower for a type 2, depending on the parameters, but certainly lower for a type 1) and dominated by symmetric information when $P > \frac{1}{2}$ (either no one is served or only type 2 is served).

When there is asymmetric information about the government's type, a government that can hide debt has advantages in doing so, if the price is sufficiently attractive. A government that can hide does not find it attractive to do it for every price. The price must be sufficiently higher than the repayment probability of its total loan (observed and hidden). This is so, because the

marginal cost of debt is higher than its repayment probability (since default is also costly). So the price must be sufficiently higher than the repayment probability of the loan to make the benefit of borrowing higher than its cost. This is the reason why, if a government cannot hide debt, the optimal amount of debt is the minimum necessary to satisfy the budget constraint. Since the marginal benefit of debt (the price) is its repayment probability and the marginal cost of debt is larger than its repayment probability, the optimal amount is the minimum necessary to allow the government to satisfy its budget constraint (the government only borrows, because borrowing is necessary to avoid default and enjoy next period's income).

If lenders are uncertain about the government's ability to hide debt, they are willing to incur in a loss in case the government can indeed hide debt, as long as they incur in a profit, in case the government cannot hide debt. The price is unfairly low to a well-behaved government and unfairly high to a bad government.

This asymmetric information price schedule makes hiding IC if the belief that the government is bad is lower than $\frac{1}{2}$. In this equilibrium, a hiding government achieves a higher payoff than under symmetric information if the belief about being of the bad type is not too high and/or if financing needs are low enough. Indeed, although a bad type enjoys an unfairly high price, it also has to borrow an unnecessarily high amount of B to mimic the behavior of a well-behaved type and keep its type private in this pooling equilibrium. Under a symmetric information equilibrium, a bad government optimally finances c through both B and K , whilst, in this equilibrium, it must finance c entirely through B . Since the positive effect of a higher price is dominated by the negative effect of a higher B , having to pool is costly for a type 2. Hence, the welfare of a type 2 only increases with asymmetric information if the gain from hiding at a high price is higher than the cost of pooling. Formally, this happens when:

$$K^2 \left(\frac{1}{2} - P \right) > \frac{1}{4} \sqrt{1 - 4c} - \frac{1}{4} (1 + PK) \sqrt{(1 - PK)^2 - 4c} - \frac{P^2 K^2}{4}$$

The left-hand-side is the gain from profitable hiding, which is the benefit from concealing its type. The right-hand-side is the cost of being in a pooling equilibrium, which is the price to pay to conceal its type. This inequality is satisfied if P is sufficiently small (so that the decrease in the price schedule is small) or if c is sufficiently low (so that the increase in B is small). The welfare loss of a type 1 is the right-hand-side of the inequality.

Lenders are equally well off under symmetric and asymmetric information, since they make zero expected profits in both cases. The one who loses with asymmetric information for sure is a well-behaved government, since it ceases to be served by the market if financing needs are too high and since its payoff at any given level of financing needs at which it is served is lower. The well-behaved government ends up paying a rent to a type 2 in order to be able to participate in the market. When this rent is too high, it becomes unprofitable to participate in the market.

So there are two distortions on welfare in this equilibrium: on the one hand, a type 1 pays a rent to a type 2 (pooling equilibrium). On the other, there are lost gains from trade, since a type 1 is excluded from the market for values of financing needs at which it participated in the market under symmetric information (separating equilibrium due to adverse selection). When type 1 is excluded from the market, a type 2 does not receive a rent from type 1 and it is as well off as it is under symmetric information. In this case, a type 1 loses welfare that is no one's gain.

If the belief that the government is bad is higher than $\frac{1}{2}$, this asymmetric information price schedule does not make hiding IC (the price is not sufficiently higher than the repayment probability of the loan to make borrowing payoff-increasing). In this case, there is no zero expected profits price that can induce a bad type into choosing the behavior it should choose for that price, except for a low enough price which makes it non-IR for a type 1 to borrow. This is a separating equilibrium, which only serves a type 2. In conclusion, if the belief of a bad type is high enough ($P > \frac{1}{2}$), the effects are that there is no lending for low levels of financing needs

(market breakdown) and, for high levels of financing needs, there is adverse selection in a separating equilibrium where only a bad type borrows.

When $P < \frac{1}{2}$, the binding constraint on the levels of financing needs at which the market provides lending to a well behaved government is the price schedule. Since the price schedule decreases with P and K (because P and K increase the risk of debt dilution) the higher P and the higher K , the lower the levels of financing needs at which a type 1 is still served by the market. The welfare of a type 2 in this pooling equilibrium is also decreasing in P , since it increases the cost of pooling, but, if P is sufficiently low, its welfare increases in K , since K increases the profit from hiding. These findings are summarized in Proposition 2.

Proposition 2:

- *If $P < \frac{1}{2}$, asymmetric information about the government's type induces a transfer of welfare from a type 1 government to a type 2. In this case, asymmetric information decreases the welfare of a type 1 and increases that of a type 2 only if P or c are sufficiently small.*
- *If $P > \frac{1}{2}$, asymmetric information decreases the welfare of a type 1 by ceasing to provide it with market lending and decreases the welfare of a type 2 by ceasing to provide it with market lending for low enough financing needs.*
- *Under asymmetric information, when $P < \frac{1}{2}$, a type 1's welfare is decreasing in P and K , but a type 2's welfare is decreasing in P and increasing in K if P is sufficiently low.*

4. Chapter II: a mechanism of costly inspection and penalties for cheating

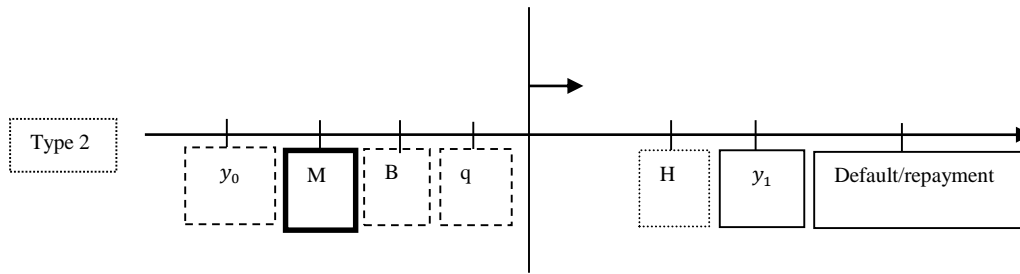
4.1 The model with the mechanism

In this section, I allow for inspection to be conducted by lenders through the payment of a fixed monitoring cost M and assume that inspection reveals the amount of hidden borrowing. If a government is found cheating, that is, if $H = K$, it has to pay a fixed penalty x to lenders.

Since I interpret this mechanism as an official bailout, the zero expected profit requirement is not imposed by perfect competition, but rather by official lenders setting a fair return to capital. I will only focus on the problem of a type 2 government and on the problem of lenders. A type 1 government cannot hide, so its problem is the same as without the mechanism. Using the timeline of the game, I start in the moment where a type 2 government chooses to hide or not and use backward induction to compute the equilibrium of the market.

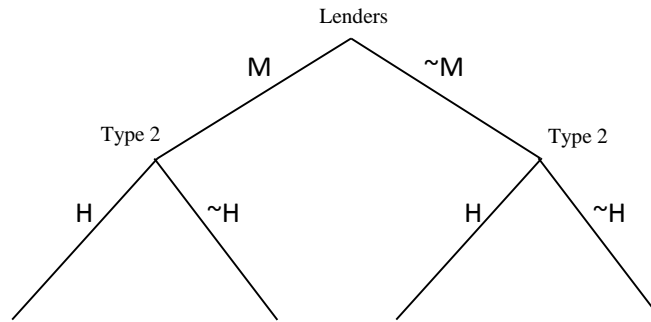
4.2 Equilibrium with commitment to inspect

When lenders make their inspection decision before granting the loan, the timeline is as follows:



A. The problem of a type 2 government

Since the government knows lenders' inspection decision when deciding whether to hide, this game is sequential and its extensive form representation is depicted below:



At the time that a type 2 government makes its hiding decision, B is a given. So, assuming that lenders decide to inspect (the left side of the tree), a type 2's expected payoff if it hides is:

$$-c + q(B + K) + \int_0^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x] dF(y_1)$$

Its expected payoff if it does not hide is:

$$-c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1)$$

Hence, a type 2 government hides if and only if the former is higher than the latter:

$$\begin{aligned} -c + q(B + K) + \int_0^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x] dF(y_1) \\ > -c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1) \end{aligned}$$

This yields the following condition on the penalty to induce a type 2 not to hide:

$$x = K \left(\frac{K}{2} - \frac{M}{B} \right)$$

On the other side, if lenders decide not to inspect (the right hand side of the tree), the problem of a type 2 government is the one under asymmetric information without the mechanism.

B. Equilibrium bond price

Assuming that the penalty induces no-hiding, lenders' payoff from inspection is:

$$E(\pi) = -qB - M + (1 - B)B$$

And the bond price schedule is pinned down by equating lenders' expected profit to 0:

$$q = 1 - B - \frac{M}{B}$$

Assuming that the penalty does not deter hiding, lenders' payoff from inspection is:

$$E(\pi) = -qB - M + Px + P(1 - B - K)B + (1 - P)(1 - B)B$$

And the bond price schedule is pinned down by equating lenders' expected profit to 0:

$$q = 1 - B - PK - \frac{Px - M}{B}$$

If lenders decide not to inspect, the equilibrium bond price is the no mechanism price schedule:

$$q = 1 - B - PK$$

C. Equilibrium lending

There are two possible equilibria by introducing the mechanism. In Equilibrium 1, lenders inspect and a type 2 government does not hide. In this equilibrium, the penalty is strong enough

to correct incentives to hide. Since it does not hide, lenders do not benefit from the penalty and must price in the monitoring cost in order to achieve zero expected profits. In Equilibrium 2, lenders inspect and a type 2 hides. In this case, the penalty is not strong enough to correct incentives to hide. However, in this case lenders extract a penalty from type 2 ex post. Hence, in Equilibrium 2, the price schedule of loans reflects not only the monitoring cost, but also a spread to cover for debt dilution risk (since a type 2 hides) and the prospect of receiving the penalty. Equilibrium 1 and 2 can coexist for the same levels of financing needs. The one that provides lending for the highest level of financing needs depends on the values of the parameters and is influenced by how the level of financing needs affects incentives to hide.

In Equilibrium 1, incentives to hide increase with the level of financing needs, while, in Equilibrium 2, they decrease. Although the first-order effects of c on the benefit and cost of hiding cancel each other out in the two equilibria (both are the probability of default), the second-order effect of c on the cost of hiding is null in both equilibria, but its effect on the benefit of hiding is positive in Equilibrium 1, since c decreases the negative effect of M on the price and it negative in Equilibrium 2, since c decreases the positive effect of the inspection profit on the price. As such, Equilibrium 2 holds only if financing needs are low, since for high levels of financing needs, hiding becomes unprofitable for a type 2, which is not an equilibrium strategy. In turn, when the inspection cost and the penalty are both low, Equilibrium 1 holds also only for low levels of financing needs, since for high levels of financing needs hiding becomes profitable. As a result, Equilibrium 1 provides lending for higher values of financing needs provided that the penalty and the inspection cost are not too low, except when the penalty is sufficiently high and the inspection cost is sufficiently low, since the price under Equilibrium 2 would benefit substantially from the expected profit from inspection. The remainder of this sub-section is technical and describes both equilibria.

Equilibrium 1's (q, B, H) is the solution to:

$$\max_B y_0 - c + q * (B) + \int_0^B 0 dF(y_1) + \int_B^1 (y_1 - (B)) dF(y_1)$$

s. t.

$$qB \geq c$$

$$q(B) = 1 - B - \frac{M}{B}$$

Both government types borrow:

$$B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(M + c)}$$

And a type 2 chooses $H = 0$.

Introducing the equilibrium amount of B on the constraint that the penalty deters hiding at the prevailing market price implies that the following condition must be satisfied in equilibrium:

$$2MK > \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(M + c)}\right)(K^2 - 2x)$$

It can be shown that if $M > -\frac{x}{2K} + \frac{K}{4}$, this condition is always satisfied. Otherwise, it imposes the following constraint on c :

$$c < \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2}\right)^2$$

Therefore, if $M > -\frac{x}{2K} + \frac{K}{4}$, the constraint that the penalty is deterrent of hiding does not bind.

In that case, since a type 2 does not hide in equilibrium, its payoff matches that of a type 1 and it is IR to borrow insofar as the bond market allows default in $t = 0$ to be avoided. Hence, the binding constraint on c is the price schedule. At the schedule $q = 1 - B - \frac{M}{B}$, the function qB attains a maximum of $\frac{1}{4} - M$, so lending exists for $c < \frac{1}{4} - M$.

On the contrary, if $M < -\frac{x}{2K} + \frac{K}{4}$, the constraint that hiding is deterred is binding and it imposes a tighter ceiling on c : $c < \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2}\right)^2$. The intuition for this constraint to bind for

$M < -\frac{x}{2K} + \frac{K}{4}$ is the following: in this equilibrium, hiding is deterred both by M (which decreases

the benefit of hiding through the price) and by x (which increases the cost of hiding). Since incentives to hide are increasing in c under the commitment mechanism, if both M and x are high enough ($M > \frac{-x}{2K} + \frac{K}{4}$), the inspection cost and the penalty are high enough to make hiding unprofitable for any positive value of c (there is no value of c that makes the benefit of hiding higher than its cost). When M and x are low ($M < \frac{-x}{2K} + \frac{K}{4}$), the price and the penalty are not high enough to make hiding unprofitable for any c . Only for $c < \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2}\right)^2$ is the cost of hiding higher than its benefit for x and M such that $M < \frac{-x}{2K} + \frac{K}{4}$.

In Equilibrium 2, lenders inspect, but the penalty does not deter hiding, so a type 2 government hides. In that case, the equilibrium (q, B, H) is the solution to:

$$\max_B y_0 - c + q * (B + K) + \int_0^B (0 - x) dF(y_1) + \int_B^1 (y_1 - (B + K + x)) dF(y_1)$$

$$s. t.$$

$$qB \geq c$$

$$q = 1 - B - PK - \frac{Px - M}{B}$$

Both government types borrow:

$$\frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)}$$

And a type 2 chooses $H = K$.

Introducing the equilibrium amount of B on the constraint that the penalty does not deter hiding at the prevailing market price implies that the following condition must be satisfied:

$$c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$$

The intuition for this upper bound on c is that, in this equilibrium, incentives to hide are decreasing in c , since the second order effect of c on the price (through B) is negative. That is

the case, because a higher c decreases the per unit (of B) profit that comes from extracting a penalty from type 2. Therefore, since a type 2 hides in equilibrium, c cannot be too high.

Moreover, if $Px > M$, the following condition must be met: $x > K^2 \frac{1-2P}{2}$. If $Px < M$, the following condition must be satisfied: $x < K^2 \frac{1-2P}{2}$.

Without inspection, the lending equilibrium is the one without the mechanism. Existence of Equilibria 1 and 2 are summarized in Lemma 4:

Lemma 4: *With a mechanism in which lenders commit to inspect the government, lending exists in equilibrium for $c < c'$ for both government types, where:*

- $c' = \frac{1}{4} - M$ if $M > \frac{K}{4} - \frac{x}{2K}$
- $c' = \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4KM}{2x - K^2} \right)^2$ if $M < \frac{K}{4} - \frac{x}{2K}$

in an equilibrium in which lenders inspect and a type 2 government does not hide (Equilibrium 1)

- $c' = \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ if $M < Px$ and $x > \frac{K^2}{2}(1 - 2P)$ or if $M > Px$ and $x < \frac{K^2}{2}(1 - 2P)$

in an equilibrium in which lenders inspect and a type 2 government hides (Equilibrium 2)

Tables 4 and 5 describe the market equilibrium as a function of c .

Table 4: Market equilibrium 1 under commitment mechanism

Information structure	Asymmetric (Hidden Type) – EQUILIBRIUM 1			
Inspection cost	$M > -\frac{x}{2K} + \frac{K}{4}$		$M < -\frac{x}{2K} + \frac{K}{4}$	
c (funding needs)	$c < \frac{1}{4} - M$	$c > \frac{1}{4} - M$	$c < \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2} \right)^2$	$c > \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2} \right)^2$
Equilibrium Borrowing	$B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(M + c)}$	$B = 0$	$B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(M + c)}$	$B = 0$

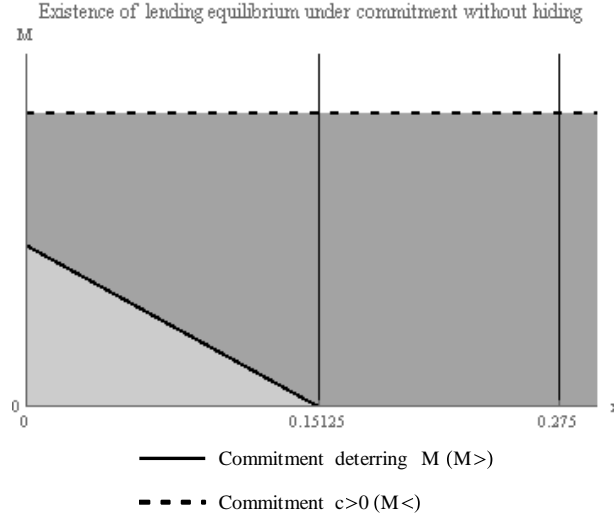
Equilibrium price schedule	$q = 1 - B - \frac{M}{B}$	$q = 0$	$q = 1 - B - \frac{M}{B}$	$q = 0$
Optimal Hidden borrowing	0	0	0	0
Expected payoff Government	$\frac{1}{4} + \frac{1}{4}\sqrt{1 - 4(M + c)} - \frac{c}{2} - \frac{M}{2}$	0	$\frac{1}{4} + \frac{1}{4}\sqrt{1 - 4(M + c)} - \frac{c}{2} - \frac{M}{2}$	0
Lenders' expected profit	0	0	0	0

Table 5: Market equilibrium 2 under commitment mechanism

Information structure	Asymmetric (Hidden Type) – EQUILIBRIUM 2				
Prior beliefs	$P > \frac{1}{2}$		$P > \frac{1}{2}$		
Inspection cost	$M < Px$	$M > Px$	$M < Px$ and $x > \frac{K^2}{2}(1 - 2P)$	$M > Px$ and $x < \frac{K^2}{2}(1 - 2P)$	elsewhere
c (funding needs)	$c < c'$	Any	$c < c'$	$c < c'$	Any
Equilibrium Borrowing	$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)}$	$B = 0$	$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)}$	$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)}$	$B = 0$
Equilibrium price schedule	$q = 1 - B - PK - \frac{M}{B} + Px$	$q = 0$	$q = 1 - B - PK - \frac{M}{B} + Px$	$q = 1 - B - PK - \frac{M}{B} + Px$	$q = 0$
Optimal Hidden borrowing	Type 2: $H = K$	$H = 0$	Type 2: $H = K$	Type 2: $H = K$	$H = 0$
Expected profit Government	Type 1: $\frac{1}{4} + \frac{1}{4}(1 - PK)\sqrt{(1 - PK)^2 - 4c + 4(Px - M)} + \frac{P^2K^2}{4} - \frac{c}{2} + \frac{Px - M}{2}$ Type 2 = Type 1 + $K(Px - \frac{M}{B} + K(\frac{1}{2} - P))$	0	Type 1: $\frac{1}{4} + \frac{1}{4}(1 - PK)\sqrt{(1 - PK)^2 - 4c + 4(Px - M)} + \frac{P^2K^2}{4} - \frac{c}{2} + \frac{Px - M}{2}$ Type 2 = Type 1 + $K(Px - \frac{M}{B} + K(\frac{1}{2} - P))$	Type 1: $\frac{1}{4} + \frac{1}{4}(1 - PK)\sqrt{(1 - PK)^2 - 4c + 4(Px - M)} + \frac{P^2K^2}{4} - \frac{c}{2} + \frac{Px - M}{2}$ Type 2 = Type 1 + $K(Px - \frac{M}{B} + K(\frac{1}{2} - P))$	0

Lenders' expected profit	0	0	0	0	0

Figure 3: Parameter space of existence of commitment mechanism equilibrium 1



In the graph of Figure 3, K and P are fixed, M varies along the vertical axis and x varies along the horizontal axis. The shaded areas in the figure are the parameter values for which a lending equilibrium exists for a non-empty set of positive values of c . Different shades indicate different upper bounds on the set of values of c for which the lending equilibrium exists.

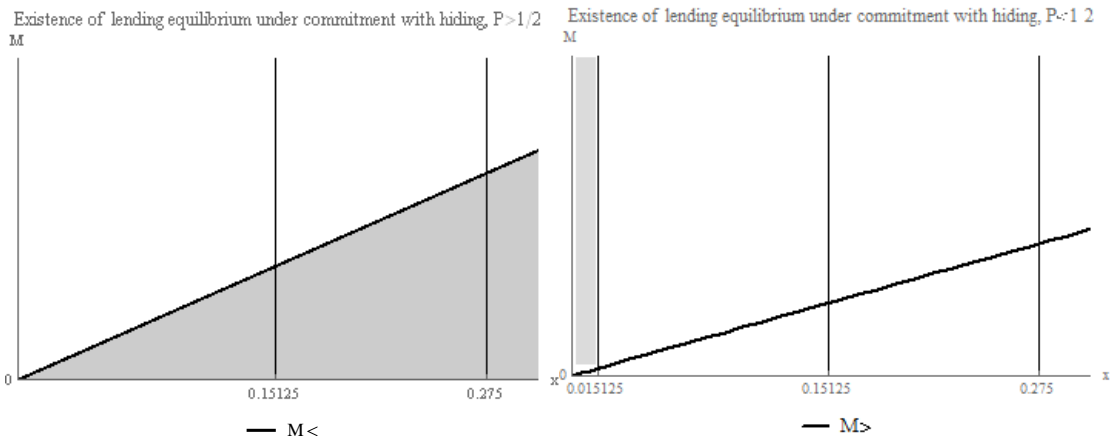
In the dark gray region, lending exists for $c < \frac{1}{4} - M$. In the light gray region, lending exists for $c < \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4MK}{2x - K^2}\right)^2$. In the no-shade region, the set of positive values of c for which lending exists is empty.

The upper bounds on c in each shaded region are determined by the constraint on c that binds in that region. In the dark gray region, the set of sustainable c is $c < \frac{1}{4} - M$, because the binding constraint on the highest sustainable c is the price schedule. So, in the dark gray region, the highest sustainable c is negative for $M > \frac{1}{4}$, it is 0 at $M = \frac{1}{4}$ and it monotonically increases as M decreases, until $M = 0$, where the highest sustainable c is $\frac{1}{4}$. The derivative of the price with

respect to the inspection cost is negative. If the inspection cost is too high, the spread charged for default risk plus the pricing in of the inspection cost per unit of debt are higher than the face value of debt and the price is negative. As the inspection cost decreases, the price increases, which increases the maximum of qB (price effect).

In the light gray region, lending exists for $c < \frac{1}{4} - M - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$, because the constraint that hiding is deterred becomes binding. So, in the light gray region, the highest sustainable c is 0 when $M = \frac{1}{4}$. As M decreases, the maximum sustainable c increases monotonically with M , until M reaches $M = \frac{-x}{2K} + \frac{K}{4}$ (the full line in Figure 3), where it discontinuously decreases from $c = \frac{1}{4} + \frac{x}{2K} - \frac{K}{4}$ to $c = \frac{1}{4} + \frac{x}{2K} - \frac{K}{4} - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$. As M decreases, $c = \frac{1}{4} - M - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$ does not vary monotonically with M . On the one hand, a lower M increases the price schedule, which raises the funding ability at the prevailing price schedule (price effect). On the other hand, a lower M decreases the deterring effect of M , which lowers the maximum c for which hiding is deterred (incentives-to-hide effect). As a result, $c = \frac{1}{4} - M - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$ first increases as M decreases (price effect dominates), but, from $M = -\frac{x}{2K} + \frac{K}{4} - \frac{(2x-K^2)^2}{8K^2}$ downwards, it decreases with M (incentives-to-hide effect dominates). When M reaches 0, $c = \frac{1}{4} - M - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$ is 0, so lending is not restored for any positive c .

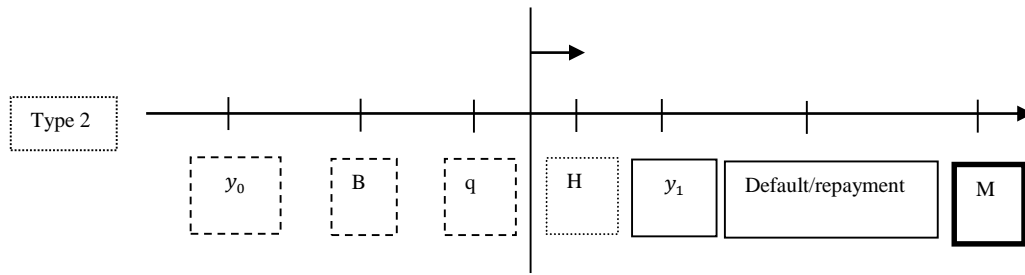
Figure 4: Parameter space of existence of commitment mechanism Equilibrium 2



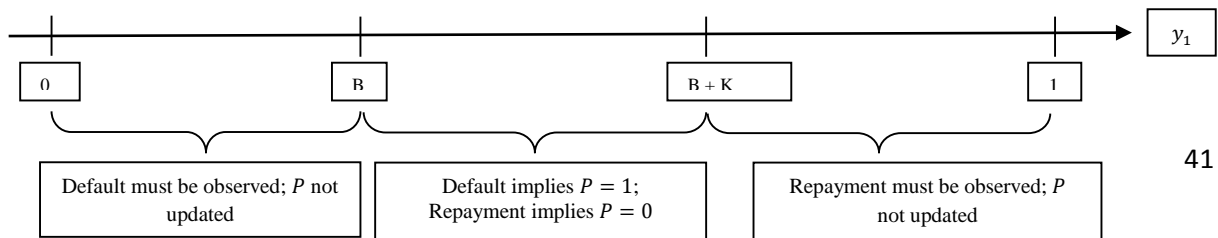
In the graphs of Figure 4, K and P are fixed, M varies along the vertical axis and x varies along the horizontal axis. In Equilibrium 2, the binding constraint on c is always the constraint that hiding is not deterred. Hence, there is only one shade of gray. So in the shaded regions of the graphs, lending exists for $c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ and in the no-shade regions, the set of positive values for which lending exists in the equilibrium is empty. When $Px > M$ (left-hand-side graph), the penalty must satisfy $x > K^2 \frac{1-2P}{2}$ (which is always true if $P > \frac{1}{2}$). When $Px < M$, the penalty must verify $x < K^2 \frac{1-2P}{2}$.

4.3 Equilibrium with no commitment to inspect

Now, I assume that lenders make their inspection decision after having observed both the realization of output and the outcome of the debt contract. The timeline is summarized below:



Given that lenders observe both the realization of y_1 and the outcome of the contract, they can infer the type 2's hidden action when y_1 is realized in the interval $[B, K]$ since the two observables form a perfectly informative signal in that region of y_1 : if $H = K$, default is optimal when $y_1 \in [B, K]$, while if $H = 0$, repayment is optimal. Hence, the observation of default leads lenders to update their belief about hiding to 1, while the observation of repayment leads them to update their belief to 0. Any other realization of y_1 is uninformative about the probability of hiding, since hiding and no-hiding lead to the same contract outcome: both would lead to default if $y_1 < B$ and both would lead to repayment if $y_1 > B + K$. So lenders cannot update their prior belief about hiding (P). The posterior beliefs about hiding as a function of y_1 are the following:



A. The problem of a type 2 government

Now the government does not know lenders' inspection decision. Lenders' expected payoff from inspection depends on whether they can update their prior belief about hiding or not. If they can, the expected revenue from inspection is x if they update P to 1 and 0 if they update P to 0. If they cannot, the expected revenue from inspection is Px . Since there is no commitment to inspect ex ante, lenders inspect if and only if it is optimal to do so.

These two possible levels of expected revenue from inspection give rise to two possible inspection strategies: if M is such that $Px > M$, the government anticipates that it is profitable for lenders to inspect for every realization of y_1 . If $Px < M < x$, the government anticipates that lenders' strategy is to inspect if and only if default is observed for $B < y_1 < B + K$.

Assuming lenders' strategy is to inspect if they observe default only if $B < y_1 < B + K$ (partial inspection rule), the expected payoff from hiding to a type 2 is:

$$-c + q(B + K) + \int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1)$$

The expected payoff from not hiding is:

$$-c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1)$$

So a type 2 government optimally hides if and only if:

$$\begin{aligned} & -c + q(B + K) + \int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1) \\ & > -c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1) \end{aligned}$$

No-hiding is a best response to partial inspection if and only if:

$$x > \frac{K}{2}$$

If lenders strategy is to inspect for every realization of y_1 (full inspection rule), the expected payoff from hiding to a type 2 is:

$$-c + q(B + K) + \int_0^{B+K} (0 - x)dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x]dF(y_1)$$

The expected payoff from not hiding is:

$$-c + q(B) + \int_0^B (0)dF(y_1) + \int_B^1 [y_1 - (B)]dF(y_1)$$

So a type 2 government optimally hides if and only if:

$$\begin{aligned} -c + q(B + K) + \int_0^{B+K} (0 - x)dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x]dF(y_1) \\ > -c + q(B) + \int_0^B (0)dF(y_1) + \int_B^1 [y_1 - (B)]dF(y_1) \end{aligned}$$

B. Equilibrium bond price

The equilibrium bond price schedule is pinned down by the following condition.

$$E(\pi) = 0$$

If $x > \frac{K}{2}$ and $x > M$, the strategies of partial inspection and no-hiding form an equilibrium. The equilibrium bond schedule is therefore:

$$q = 1 - B$$

The penalty is so severe that the prospect of being inspected with probability K (which is the probability of $B < y_1 < B + K$, which is the region where lenders infer the hiding choice of a type 2) is enough to make it optimal for a type 2 not to hide. The expected cost of hiding is higher than its benefit. So the price that achieves zero expected profits does not need to reflect the risk of hiding nor the cost of inspecting, nor the probability of a type 2.

If $x < \frac{K}{2}$, this equilibrium does not hold, since a type 2 would optimally hide. However, if lenders increase the frequency of inspection to full inspection (which they optimally do as long as $Px > M$), hiding may be deterred. If hiding is not deterred, there exists an equilibrium equal to Equilibrium 2 under commitment to inspect. If hiding is deterred, lenders lose incentives to inspect, since they update P to 0 and inspecting becomes unprofitable. However, if they do not

inspect, it becomes profitable for a type 2 to hide. So there is no pure strategies equilibrium with no-hiding by a type 2 and inspection by lenders, as it does under commitment. However, since lenders' strategy is binary (inspect/not inspect) and a type 2's strategy is also binary (hide (K)/not hide), a mixed strategies equilibrium exists.

If $Px > M$, it is profitable to inspect when lenders cannot infer hiding and when they can. Assuming that lenders inspect with probability γ when they cannot infer hiding (that is, when $y_1 < B$ or $y_1 > K$), a type 2's payoff from hiding is:

$$\begin{aligned} & -c + q(B + K) + \gamma \left(\int_0^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x] dF(y_1) \right) \\ & + (1 - \gamma) \left(\int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) \right. \\ & \left. + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1) \right) \end{aligned}$$

A type 2's payoff from not hiding is:

$$c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1)$$

So a type 2 government is indifferent between hiding and not hiding if and only if:

$$\begin{aligned} & -c + q(B + K) + \gamma \left(\int_0^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x] dF(y_1) \right) \\ & + (1 - \gamma) \left(\int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) \right. \\ & \left. + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1) \right) \\ & = -c + q(B) + \int_0^B (0) dF(y_1) + \int_B^1 [y_1 - (B)] dF(y_1) \end{aligned}$$

Which yields:

$$\gamma = \frac{1}{x(1 - K)} K \left(-P\rho K + \frac{K}{B} P\rho(x - M) + \frac{K}{2} - x \right)$$

In turn, assuming that a type 2 government hides with probability ρ , lenders are indifferent between inspecting and not inspecting when $y_1 < B$ or $y_1 > K$ if and only if:

$$P\rho x = M$$

So, as long as $Px > M$, the equilibrium price schedule in this mixed strategies equilibrium is:

$$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$$

This price schedule differs from the one prevailing in the high penalty equilibrium, with the presence of hiding risk pushing the price down and the presence of profitable inspection pushing the price up.

C. Equilibrium lending

Under no commitment, there are three possible lending equilibria. In Equilibrium 1, lenders do not inspect in equilibrium and a type 2 does not hide. In this equilibrium, the penalty is so high that it corrects incentives to hide. Moreover, it does not waste resources on inspection, since the government anticipates that it will indeed be optimal for lenders to inspect, so no commitment to inspect is necessary. Therefore, ex ante, lenders offer the first-best price. Equilibrium 2 with commitment also holds here, but only when $Px > M$, since lenders need incentives to inspect). The price is damaged by the risk of dilution, since a type 2 hides as well as by the cost of inspection but reflects the expected penalty to be extracted from a type 2. In Equilibrium 3, lenders inspect for sure when a type 2 reveals its type and inspect with probability γ in all other ex post circumstances. In turn, a type 2 government hides with probability ρ . Given that lenders need to have incentives to inspect ex post, there is no equilibrium that eliminates hiding risk, since a type 2 would need the threat of inspection to lose the incentive to hide, but it would not be IC for lenders to inspect if they anticipate no hiding. In this equilibrium, the price reflects some debt dilution risk and, additionally, the profit from inspection that lenders expect when they can update the belief about a type 2 to 1.

Equilibria 1 and 3 cannot coexist for any given level of financing needs, because, if the penalty is high enough to implement Equilibrium 1, Equilibrium 1 is necessarily implemented since it is incentive compatible for lenders ex post. On the contrary, Equilibrium 2 and 3 can coexist for some levels of financing needs. In both equilibria, incentives to hide decrease with c , due to the negative second-order effect of c on the price by decreasing the per unit profit from inspection. The one that provides lending for the highest c depends on the parameter values. The remainder of this sub-section is technical and describes all equilibria.

If $x > \frac{K}{2}$, the equilibrium amounts of borrowing for a type 1 and a type 2 are both:

$$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$$

And a type 2 does not hide. In this equilibrium (Equilibrium 1), a type 2 does not hide and lenders do not inspect (since they do not have ex post evidence to do so).

If $x < \frac{K}{2}$, Equilibrium 2 under the commitment mechanism also exists without commitment, but only when $Px > M$, since when $Px < M$ lenders do not have incentives to inspect, which they need if they do not commit. I call this Equilibrium 2 under no commitment.

If $x < \frac{K}{2}$, the equilibrium amount of B for both government types under the mixed strategies equilibrium (Equilibrium 3) is:

$$B = \frac{1}{2x}(x - KM) - \frac{1}{2x}\sqrt{(x - KM)^2 + 4x(KM(x - M) - xc)}$$

And the equilibrium amount of H for a type 2 government is:

$$H = \rho K$$

In order for ρ and γ to be meaningful, their values must be between 0 and 1. ρ is always positive and the constraint that it is lower than 1 imposes $M < Px$. The intuition is that, for $M > Px$, there is no level of ρ below 1 that leaves lenders indifferent between inspecting and not inspecting. Since lenders must be indifferent between inspecting and not inspecting when $B < y_1 < K$, a type 2 government would have to hide with probability higher than 1 in order to make

lenders indifferent between inspection and no inspection, which is meaningless. Since ρ would necessarily be lower than the necessary value to equate the expected penalty with the inspection cost, lenders would optimally not inspect, which is not an equilibrium strategy of the lending equilibrium.

The requirement that ρ is between 0 and 1 does not impose constraints on c , since ρ is defined as the probability of hiding by a type 2 that leaves lenders indifferent between inspecting and not inspecting, which is independent of the amount borrowed.

In turn, the requirement that γ is between 0 and 1 does impose restrictions on c , since γ is defined as the probability of inspection that leaves a type 2 indifferent between hiding and not hiding and that trade off depends on c , through the impact that c has on B and the impact that B has on the benefit of hiding (determined by the price) and on the cost of hiding (determined by the default probability).

For γ to be positive, either the inspection cost and the penalty are low enough ($M < \max\{M_2, M_3\}$), or c must be low enough ($c < c^{**}$). The intuition for this upper bound on c is the following: in this equilibrium, a type 2 must be indifferent between hiding and not hiding. Since the cost of hiding increases with γ and incentives to hide are decreasing in c , γ must be such that the benefit of hiding equals the cost of hiding, for a given c . Since incentives to hide are decreasing in c , there is an upper value on c for which the cost of hiding would be equal to the benefit of hiding through a positive value of γ . Above such an upper bound, the benefit of hiding is so low that γ would have to decrease to negative values in order to decrease the cost of hiding up to the point of indifference between hiding and no hiding.

The constraint that γ is positive does not bind if $M < \max\{M_2, M_3\}$, because M becomes so low that the benefit of hiding is very high (through the price) and γ does not need c to decrease the cost of hiding. In this case, the binding constraint on c is the price schedule. The price

schedule limits fund raising to the maximum of the function qB , which attains a maximum of c^* at the price schedule $q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$.

For γ to be lower than 1, the inspection cost and the penalty cannot both be too low (M must be above M_1) and, moreover, c cannot be too low ($c > c^{****}$). This is caused by the fact that, in equilibrium, a type 2 government must be indifferent between hiding and not hiding. Since the cost of hiding is a positive function of γ , γ must be such that the benefit of hiding equals the cost of hiding, for given (x, M) . Since, under this mechanism, incentives to hide decrease with c , if c is too low, the incentive to hide is so high that γ would have to be higher than 1 in order to raise the cost of hiding to the level of the benefit of hiding. Since γ is bounded above at 1, c has a lower bound (“incentives-to-hide effect of c ”).

Moreover, if $M < M_1$, γ is not lower than 1 for any pair (x, M) . The intuition is the following: when M reaches M_1 , both the inspection cost and the penalty are low, so the benefit of hiding is very high (the inspection cost does not push the price down by much and the penalty does not push the cost of hiding up by much). Since γ must be such that the benefit of hiding equals the cost of hiding, the cost of hiding must be high and, below M_1 , that would require γ to be higher than 1, for any possible positive value of c .

Lemma 5 summarizes the existence of these Equilibria under no commitment:

Lemma 5: *With a mechanism in which lenders do not commit to inspect, lending exists in equilibrium for both government types for $c < c'$, where:*

- $c' = \frac{1}{4}$ if $M < x$
- $c' = 0$ if $M > x$

If $x > \frac{K}{2}$, in an equilibrium in which lenders do not inspect and a type 2 does not hide (Equilibrium 1)

- $c' = \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ if $M < Px$ and

$$x > \frac{K^2}{2} (1 - 2P) \text{ (condition on } x \text{ is always satisfied if } P > 1/2).$$

- $c' = 0$ if $M > Px$

If $x < \frac{K}{2}$, in an equilibrium in which lenders inspect and a type 2 government hides (Equilibrium

2)

And lending exists in equilibrium for both government types for $c'' < c < c'$, where:

- $c' = c'' = 0$ if $M < \max\{M_1, M_5\}$
- $c' = c^*$ and $c'' = c^{****}$ if $\max\{M_1, M_5\} < M < \max\{M_2, M_3\}$
- $c' = c^{***}$ and $c'' = c^{****}$ if $\max\{M_2, M_3\} < M < Px$
- $c' = c'' = 0$ if $M > Px$

If $x < \frac{K}{2}$, in an equilibrium in which lenders inspect with probability γ and a type 2 government

hides with probability ρ (Equilibrium 3).¹⁰

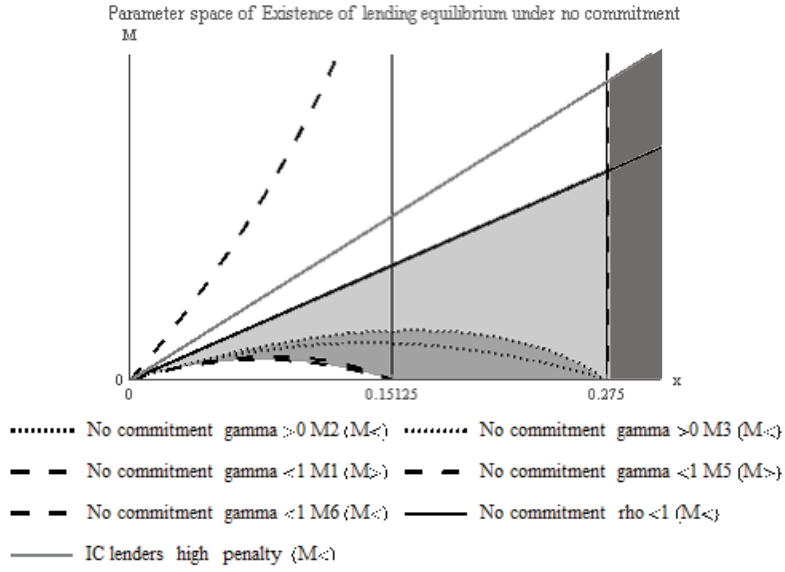
Table 6: Market equilibria 1 and 3 under no commitment mechanism

Information structure	Asymmetric (Hidden Type) – mixed strategies equilibrium					
Penalty	$x > \frac{x}{2}$		$x < \frac{x}{2}$			
Inspection cost	Any	Any	$\max\{M_1, M_5\} < M < \max\{M_2, M_3\}$		$\max\{M_2, M_3\} < M < Px$	
c (legacy bills)	$c < \frac{1}{4}$	$c > \frac{1}{4}$	$c < c^*$		$c > c^*$	$c < c^{***}$
Equilibrium B	$B = \frac{1}{2} - \frac{1}{2}\sqrt{1-4c}$	$B = 0$	$B = \frac{1}{2x}(x - KM) - \frac{1}{2x}\sqrt{(x - KM)^2 + 4x(KM(x - M) - xc)}$		$B = 0$	$B = \frac{1}{2x}(x - KM) - \frac{1}{2x}\sqrt{(x - KM)^2 + 4x(KM(x - M) - xc)}$
Equilibrium price schedule	$q = 1 - B$	$q = 0$	$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$		$q = 0$	$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$
Optimal H	0	0	$P\rho K$		0	$P\rho K$

¹⁰ The explicit expressions of c' and c'' in Equilibrium 3 are in the Appendix.

Expected payoff Government	$\frac{1}{4} + \frac{1}{4}\sqrt{1-4c} - \frac{c}{2}$	0	Type 1 = Type2: $\frac{1}{4} + \frac{1}{4x}\sqrt{(x-KM)^2 + 4x(KM(x-M) - xc)} - \frac{c}{2} + \frac{KM}{2}(1 - \frac{M}{x} + \frac{KM}{2x} + \frac{\sqrt{(x-KM)^2 + 4x(KM(x-M) - xc)}}{2x})$	0	Type 1 = Type2: $\frac{1}{4} + \frac{1}{4x}\sqrt{(x-KM)^2 + 4x(KM(x-M) - xc)} - \frac{c}{2} + \frac{KM}{2}(1 - \frac{M}{x} + \frac{KM}{2x} + \frac{\sqrt{(x-KM)^2 + 4x(KM(x-M) - xc)}}{2x})$	0
Expected profit lenders	0	0	0	0	0	0

Figure 5: Parameter space of existence of no commitment mechanism Equilibria 1 and 3



The shaded areas are the parameter regions where a lending equilibrium exists for a non-empty interval of positive values of c . The bounds of that interval differ with the intensity of the shade.

In the darker gray region, lending is restored for $c < \frac{1}{4}$. In the medium-dark gray region, lending is restored for $c^{***} < c < c^*$, while, in the light gray region, lending is restored for $c^{***} < c < c^{**}$. In the non-shaded regions, lending is restored for the empty set.

The bounds of the interval of c values for which lending exists are determined by which constraints on c imposed by the prevailing equilibrium bind in each parameter region. In the dark-gray region, ($x > \frac{K}{2}$), the market is in a lending equilibrium in which the government does not hide and lenders do not inspect. The binding constraint on c is the price schedule, which imposes that lending exists for $c < \frac{1}{4}$. In addition, the inspection cost M must be lower than the

penalty x (the gray full line in Figure 5), so that lenders' strategy (the rule of inspecting if and only if they see default when $B < y_1 < K$) is incentive compatible.

In the light gray region, the market is in the mixed strategies equilibrium. Since in this region $M > \max\{M_2, M_3\}$, the binding constraint on the upper bound on c is the constraint that γ is positive, which is c^{***} . The constraint that γ is lower than 1 imposes a lower bound on c , which is c^{****} . Therefore, in this region, lending exists for $c^{****} < c < c^{***}$.

In the medium-dark gray region, the market is also in the mixed-strategies equilibrium. Since $M < \max\{M_2, M_3\}$ in this region, the constraint that γ is positive is not binding. So lending exists for $c^{****} < c < c^*$. The lower bound on c is the same as in the light gray region, because the constraint that γ is lower than 1 binds in the entire region below the black full line. The upper bound on c results from the fact that the binding constraint is now the price schedule.

In the no shade region above the black full line ($M > Px$), the set of positive values of c for which lending exists is empty, because the market is in the mixed strategies equilibrium and the constraint that ρ is lower than 1 is not satisfied for any pair (x, M) if $M > Px$.

In the no-shade region below the lower dashed parabola ($\max\{M_1, M_5\}$), the set of sustainable c is empty, because the constraint that γ is lower than 1 is not satisfied for any pair (x, M) .

In summary, starting at $M = Px$, the maximum sustainable level of c is c^{***} . As M reaches $\max\{M_2, M_3\}$, the upper bound on c increases discontinuously to c^* , which holds unless M reaches $\max\{M_1, M_5\}$, where lending is not provided for any c in this equilibrium. For example, by taking a vertical line starting at $M = Px$ and moving downwards, a given value of c that is not sustained in a lending equilibrium in the light gray region becomes sustainable in the dark gray one, because, although the penalty is the same, M is lower, which increases the price, which increases the benefit of hiding, which increases the highest value of c for which a positive γ equates the marginal cost with the marginal benefit, given that incentives to hide decrease

with c . The market may also be in Equilibrium 2, if $Px > M$, in which case the mechanism provides lending for $c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$.

Lemma 6: *There exists a set of parameters x , M , K and P for which c'' in Equilibrium 3 is higher than c' in Equilibrium 2.*

5. Chapter III: Who and when benefits from the introduction of a mechanism?

A mechanism implements lending independently of the degree of asymmetric information (P), by controlling the behavior of a type 2 government. Controlling the behavior of a type 2 government has both a negative and a positive externality on type 1. The negative externality is the cost of inspection, which is paid not only by the bad type but also by the well-behaved type. This decreases the mechanism price relative to the “no mechanism” price. The positive externality is the penalty, which, under some mechanism equilibria, is extracted from a type 2 and given to lenders, but which ends up benefitting the good type, since perfect competition induces it to be priced in. So the penalty allows for some of the benefit from hiding to be shared between the two types. This pushes the mechanism price up relative to the “no mechanism” price.

5.1 Does the welfare of a type 1 improve?

Starting by the case when a mechanism certainly improves welfare for a type 1 ($P > \frac{1}{2}$), since without a mechanism it is not served by the market for any level of financing needs, the mechanism that restores lending for the highest c depends on the extent to which the mechanism reduces hiding risk and on the strength of these two externalities.

On the one hand, the commitment mechanism, entails very frequent inspection. With the no commitment mechanism, the mixed strategies equilibrium entails less frequent inspection, so the price suffers a lower haircut due to this negative externality. On the other hand, the commitment mechanism is able to eliminate hiding risk. The no commitment mechanism is

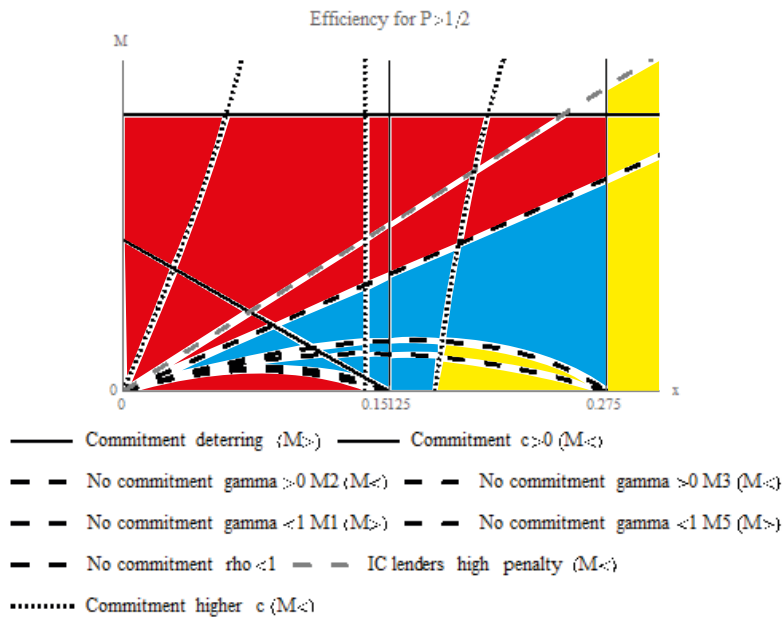
unable to completely eliminate hiding risk, so the price is negatively affected the presence of hiding risk. Finally, both mechanisms are able to set up an equilibrium with frequent hiding, so both are able to let a type 2 benefit from the positive externality coming from the penalty. Lemma 7 describes which mechanism provides lending for the highest c for the parameters.

Lemma 7: *The inspection mechanism that implements lending for the highest c is:*

- Commitment, if $0 < x < \frac{K^2}{2}$
- Commitment, if $\frac{K^2}{2} < x < \frac{K}{4}$ and $M < \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}$
- Commitment, if $\frac{K}{4} < x < \frac{K}{2}$ and $M > \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}$
- No commitment, if $x > \frac{K}{2}$.

Figure 6 portrays Lemma 7 in the (x, M) space. In the red regions a lending equilibrium exists only with the commitment mechanism. In the blue ones, lending is implemented by both mechanisms, but it is so for higher values of c under the commitment mechanism. In the yellow areas, on the contrary, lending in equilibrium exists under both mechanisms, but it exists for higher values of c under the no commitment mechanism.

Figure 6: Inspection mechanism that implements lending for the highest c



For $0 < x < \frac{K^2}{2}$, the commitment mechanism restores lending for the highest value of c either because the market is in the mixed strategies equilibrium under no commitment, in which case only the commitment mechanism is able to restore lending (red part below the bottom parabola), or because the market is in the common equilibrium under no commitment (Equilibrium 2), in which case either the commitment mechanism restores lending for the same values of c (since it can implement the same equilibrium) or the commitment mechanism restores lending for higher values of c , by implementing the no-hiding equilibrium (blue part), which restores lending for higher values of c than the common equilibrium if the inspection cost is high enough. The intuition is that those two equilibria inspect with the same frequency, so the strength of the negative externality caused by inspection is the same, and in the case of the common equilibrium there is hiding risk, which damages the price. If the penalty is low relative to the price, the positive externality coming from the penalty is not enough to compensate for the presence of hiding risk.¹¹

For $\frac{K^2}{2} < x < \frac{K}{4}$, the commitment mechanism restores lending for the highest value of c if $M < \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}$ because it can implement an equilibrium with no-hiding (Equilibrium 1), which restores lending for higher values of c than the mixed strategies equilibrium under no commitment. The intuition is that both the inspection cost and the penalty are low, so deterring hiding by inspecting very frequently does not impose a very strong negative externality on the price and forfeiting the penalty because hiding is deterred does not waste a very strong positive externality. On top of that, there is no haircut on the price due to the risk of dilution. All in all, the price achieved in the no-hiding equilibrium is higher than the price achieved in any other

¹¹ The region of parameters in which Equilibrium 1 implements lending for higher values of c than Equilibrium 2 under commitment is not depicted in Figure 6, because both if Equilibrium 1 implements lending for higher values of c than Equilibrium 2 and vice-versa, the no commitment mechanism is dominated by the commitment mechanism when Equilibrium 1 under commitment implements lending for higher values of c than Equilibrium 3 or Equilibrium 1 under no commitment. So it is the comparison between the commitment mechanism's Equilibrium 1 and no commitment mechanism's Equilibria 1 and 3 that dictates welfare.

mechanism equilibrium. In this region, the binding constraint on the no-hiding equilibrium is the price. When the binding constraint on the mixed strategies equilibrium is not the price, the constraint that binds imposes a tighter ceiling on c (either 0, in the red region above the large-dashed black line, or a positive value, in the blue region between the large-dashed black line and the top parabola), so the upper bound on c under the no-hiding equilibrium is still higher. If the market is in Equilibrium 2 under no commitment, the commitment mechanism restores lending for the same values of c as no commitment, since it can also implement Equilibrium 2. For $\frac{K}{4} < x < \frac{K}{2}$, the commitment mechanism restores lending for the highest value of c if $M > \frac{-4x^2(1+\frac{1}{K})+2x}{K-4x}$ because the no-hiding equilibrium implementable under commitment (Equilibrium 1) restores lending for higher values of c than the mixed strategies equilibrium under no commitment. The intuition is that, since the penalty is relatively high, and therefore its positive externality on the price is relatively strong, the no-hiding equilibrium only implements a higher price if the inspection cost is so high that a bad government would have to hide with a very high probability in order for lenders to be indifferent between hiding and not hiding, which is a requisite in the mixed strategies equilibrium. This damages the mixed strategies price by increasing dilution risk (blue region below the dashed parabola). If the inspection cost is not so high, then the positive externality coming from the penalty is stronger than the effect of dilution risk and the mixed strategies price is higher than the no-hiding price (yellow region below the dashed parabola). Once again, in those regions in which the binding constraint on c in the mixed strategies equilibrium is not the price, the constraint that binds imposes a tighter ceiling on c (0 in the red region above the large-dashed red line and a positive value in the blue region between the top dashed parabola and the large-dashed black line), so the no-hiding continues to dominate the mixed strategies equilibrium. If the market is in Equilibrium 2 under no commitment, the commitment mechanism restores lending for the same values of c as no commitment, since it can also implement Equilibrium 2.

Finally, for $x > \frac{K}{2}$, the no commitment mechanism restores lending for the highest value of c because it can implement the first-best price through a no-hiding/no-inspection equilibrium (yellow region on the right-hand-side of the graph).

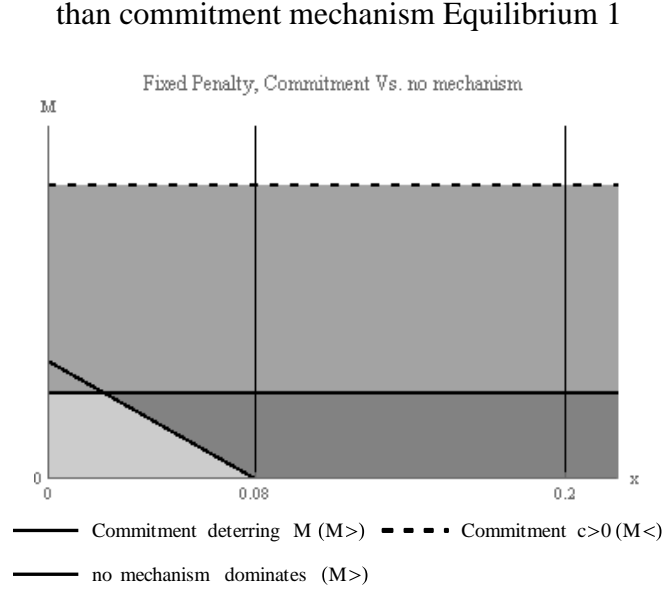
Moving on to the case when a mechanism might or might not improve the welfare of a type 1 government ($P < \frac{1}{2}$) a type 1 might be better off or worse off with the introduction of a mechanism, depending on the extent to which hiding risk is reduced with a mechanism and on the strength of the two externalities (the negative one imposed by the inspection cost and the positive one imposed by the penalty), which depends on the parameters. Indeed, the loss suffered by a good government due to asymmetric information might be higher with a mechanism than without a mechanism.

Starting by the commitment mechanism, when the market is in the no-hiding equilibrium, the negative externality imposed by inspection is very strong, since inspection occurs very frequently. If the inspection cost is too high (above $PK(\frac{2-PK}{4})$), this negative externality is stronger than the haircut imposed on the price due to debt dilution under “no mechanism”. In this case, the upper bound on c imposed by the mechanism is lower than the one imposed by “no mechanism” (which is $c = \frac{1}{4} - \frac{PK(2-PK)}{4}$). So the adverse selection effect under no mechanism, although bad for a well-behaved government, is nonetheless less harmful than the negative externality imposed by the mechanism. This is represented in the (x, M) space in Figure 7 by the gray shade above the horizontal line.

If the inspection cost is not so high (below $PK(\frac{2-PK}{4})$), then the opposite holds and introducing a mechanism is welfare-improving for a type 1 insofar as the binding constraint on c under the commitment no-hiding equilibrium is the price, which is not the case if the inspection cost and the penalty are both very low. In this case, the upper bound on c imposed by the constraint that

hiding is deterred is lower than the one under “no mechanism”. In this region of parameters (the light gray region in Figure 7), introducing a mechanism leaves a type 1 government worse off.

Figure 7: Parameter space where “no mechanism” implements lending for higher c values



When the market is in the hiding equilibrium, the negative externality imposed by inspection is very strong, since inspection occurs very frequently in this equilibrium too, but there is a positive externality coming from the penalty. On the other side, since hiding is not deterred, the mechanism price is damaged by the presence of hiding risk. So both the mechanism and the “no mechanism” prices are damaged by the presence of hiding risk. Only if the positive externality coming from the penalty is stronger than the negative externality caused by inspection, does the commitment mechanism restore lending for higher values of c than “no mechanism”. This happens if $M < M'$.

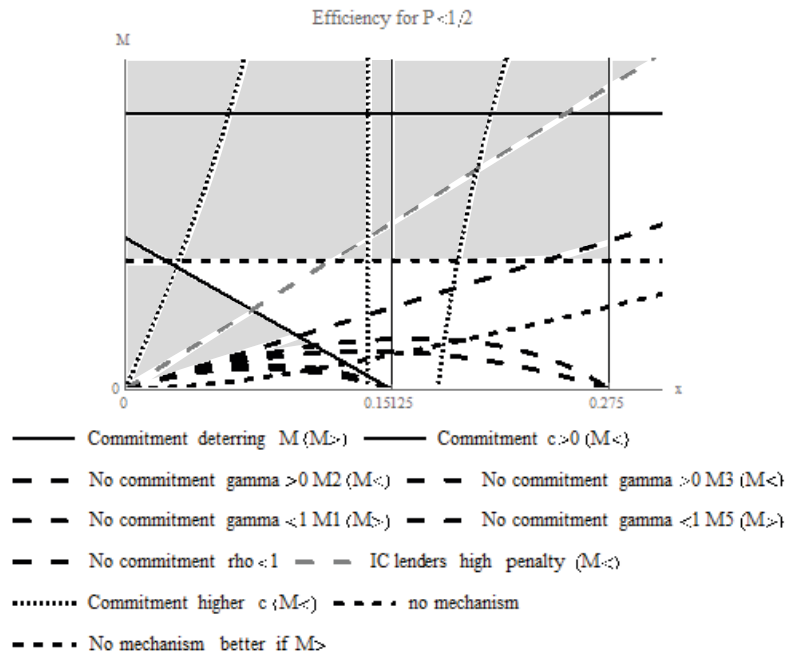
Going to the no commitment mechanism, the no commitment regime can only increase the parameter space where the introduction of a mechanism improves the welfare of a type 1 relative to the space studied in the previous paragraph in the parameter regions where it dominates the commitment regime, which happens in the yellow regions of Figure 6. Hence, only if hiding risk is so low that it damages the price so little that “no mechanism” implements lending for higher values of c than the no-hiding commitment equilibrium for relatively low

levels of the inspection cost can the no commitment mechanism increase the parameter space where the introduction of a mechanism improves the welfare of a type 1. That would require the full horizontal line of Figure 7 to be lower than the top dashed parabola of Figure 6. In that case, the no commitment mixed strategies equilibrium would attain the highest price by reducing hiding risk relative to “no mechanism” without imposing too strong a negative externality through inspection and by adding a positive externality through the penalty. Lemma 8 summarizes these findings.

Lemma 8: When $P < \frac{1}{2}$, “no mechanism” implements lending for the highest c for a type 1 government if $M > \max\{M', PK\left(\frac{2-PK}{4}\right), M_2, M_3\}$ or $\max\{M', M_2, M_3\} < M < \frac{K}{4} - \frac{x}{2K}$ when $x < \frac{K}{2}$ and if $M > \max\{M', PK\left(\frac{2-PK}{4}\right), x\}$ when $x > \frac{K}{2}$.

Figure 8 portrays the description in Lemma 8 in the (x, M) space. The gray-shaded regions are the regions of parameters in which the introduction of a mechanism actually leaves a type 1 worse off, when there is asymmetric information.

Figure 8: Space where “no mechanism” provides lending for the highest c for a type 1 ($P < \frac{1}{2}$)



In the gray region to the left of $x = \frac{K}{2}$, the introduction of a mechanism is actually harmful for a type 1, since no mechanism implements lending for a highest c for a type 1 than the no-hiding equilibrium under commitment, the hiding equilibrium under the two mechanisms and the mixed strategies equilibrium under no commitment. In the gray region to the right of $= \frac{K}{2}$, the introduction of a mechanism is harmful for a type 1, since the no-hiding/no-inspection equilibrium implementable under the no commitment mechanism is not IC for lenders. So “no mechanism” is not dominated by that additional mechanism equilibrium.

5.2 Does the welfare of a type 2 improve?

As for a type 2, the introduction of a mechanism leaves it better off only if $P > \frac{1}{2}$ and financing needs are low, since in that case there is market breakdown for both types without the mechanism. When $P < \frac{1}{2}$, the introduction of a mechanism surely cannot improve a type 2's welfare, since there is no mechanism equilibrium that serves a type 2 for higher levels of financing needs than “no mechanism”. When a mechanism equilibrium serves a type 2 for lower values of c than “no mechanism”, a type 2's welfare is unchanged, since it can always reveal its type and be as well off as under “no mechanism”.

When neither type borrows without the mechanism, the welfare of a bad government decreases as a result of asymmetric information. Then the introduction of a mechanism can improve a type 2's welfare, by restoring the market for levels of financing needs at which a type 2 is not served under “no mechanism”. Those levels are low levels of financing needs. However, since the measure of welfare is the *upper* bound on financing needs that can be financed in the market, the introduction of a mechanism does not leave a type 2 better off in this situation either, since a type 2 is still served under “no mechanism” when financing needs are high – only when they are low does the market not provide lending. Therefore, there is no parameter region in

which a type 2's welfare is increased through the introduction of a mechanism, when there is asymmetric information in the market. These findings are summarized in Proposition 3.

Proposition 3: *With asymmetric information about the government's type, a type 1's welfare increases with the introduction of a mechanism when $P > \frac{1}{2}$. When $P < \frac{1}{2}$, a type 1's welfare decreases with the introduction of a mechanism if $M > \max\{M', PK\left(\frac{2-PK}{4}\right), M_2, M_3\}$ or $\max\{M', M_2, M_3\} < M < \frac{K}{4} - \frac{x}{2K}$ and $x < \frac{K}{2}$ and increases elsewhere. A type 2's welfare increases with the introduction of a mechanism for $c < \frac{1}{4} - \frac{1}{2}K\left(1 - \frac{K}{2}\right)$ and decreases for $c > \frac{1}{4} - \frac{1}{2}K\left(1 - \frac{K}{2}\right)$, when $P > \frac{1}{2}$. When $P < \frac{1}{2}$, a type 2's welfare decreases with the introduction of a mechanism for the entire parameter space.*

5.3 Does aggregate welfare improve?

Overall, asymmetric information makes welfare incomparable when $P < \frac{1}{2}$ (either higher or lower for a type 2, depending on the parameters, but certainly lower for a type 1) and dominated by symmetric information when $P > \frac{1}{2}$ (either no one is served or only type 2 is served).

Accordingly, the introduction of a mechanism makes welfare incomparable when $P < \frac{1}{2}$ (either higher or lower for a type 1, depending on the parameters, and equal for a type 2, since a type 2 can always reveal its type and is served for as high levels of c as without any mechanism) and higher relative to “no mechanism” when $P > \frac{1}{2}$ (lending is restored for a type 1 for some levels of financing needs and a type 2 is served for as high levels of c as without any mechanism).

5 Conclusions

The first main conclusion is that the sovereign debt market is information-sensitive to the true amount of borrowing. Under asymmetric information, welfare decreases for a well behaved government, because it is not served by the market for any level of financing needs if lenders' belief that the government can hide is sufficiently high and, if that belief is not so high, there is

adverse selection for high levels of financing and each level of financing needs becomes more expensive to finance, which lowers its payoff.

The rise in Portuguese sovereign yields after Greece revealed misreported deficit data could be interpreted, in this model, as a new equilibrium, in which investors held a higher belief than before that Portugal was not transparent, which would increase yields. The model's prediction of market breakdown if the belief is too high is consistent with Portugal leaving the capital markets and requesting an external bailout.

The second main conclusion is that allowing for bailout lending, with the possibility for official lenders to inspect the accounts of the government and impose penalties in case of cheating, might decrease the welfare of a well-behaved government relative to market lending, even though the market is charging an unfairly high yield due to asymmetric information.

So even if sovereign debt yields rise as a result of uncertainty regarding the government's transparency, if the market still provides lending, the market solution might still be the best solution, when compared to requesting an official bailout on the grounds that spreads are too high due to a distortion caused by information asymmetries.

Comparative statics of the baseline model equilibrium on P and K also allow to conclude about the importance of a "reputation for transparency". The lower the P and the K , the lower the decrease caused by uncertainty about the government's type on a well-behaved government's welfare. This would recommend countries to build a reputation for transparency, so as to decrease P and K . This could be accomplished, for example, through constitutional rules or independent fiscal bodies. One possible extension of the model is to study whether rewards for no-hiding would have similar effects to penalties for hiding.

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Appendix

1. Derivation of the price schedule for a type 1 government
2. Market equilibrium for a type 1 government
3. Marginal cost and marginal benefit of borrowing
4. Market equilibrium with perfect enforcement
5. Proof of Lemma 1
6. Proof that the optimal choice of H by a type 2 is a corner solution
7. Price derivation for a type 2 government
8. Market equilibrium for a type 2 government
9. Equilibrium price under asymmetric information
10. Market equilibrium under asymmetric information
11. Proof of Lemma 3
12. Proof of Proposition 2
13. Proof of Lemma 4
14. Proof of Lemma 5
15. Proof of Lemma 6
16. Proof of Lemma 7

17. Proof of Lemma 8

1. Derivation of the price schedule for a type 1 government:

$$E(\pi) = 0$$

$$-qB + E(B) = 0$$

$$qB = E(B)$$

$$qB = B * Prob(y_1 \geq y_0 + B) + 0 * Prob(y_1 < y_0 + B)$$

$$qB = B * \left(1 - \frac{y_0 + B - y_0}{\bar{y} - y_0}\right) + 0 * \left(\frac{y_0 + B - y_0}{\bar{y} - y_0}\right)$$

$$q(B) = 1 - \frac{B}{\bar{y} - y_0}$$

$$q(B) = 1 - B$$

Debt ceiling:

$$Prob.(y_1 \geq y_0 + B) \geq 0$$

$$1 - Prob.(y_1 < y_0 + B) \geq 0$$

$$1 - F(y_0 + B) \geq 0$$

$$1 - \frac{y_0 + B - y_0}{y_1 - y_0} \geq 0$$

$$1 - \frac{y_0 + B - y_0}{y_1 - y_0} \geq 0$$

$$1 - B \geq 0$$

$$B \leq 1$$

2. Market equilibrium for a type 1 government:

Problem:

$$\max_B y_0 - c + qB + \int_{y_0}^{y_0+B} y_0 dF(y_1) + \int_{y_0+B}^{\bar{y}} (y_1 - B) dF(y_1)$$

s. t.

$$q = 1 - \frac{B}{\bar{y} - y_0}$$

$$qB \geq c - y_0$$

$$B \leq \bar{y} - y_0$$

$$\max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{1}{\bar{y} - y_0} \int_{y_0}^{y_0+B} y_0 dy_1 + \frac{1}{\bar{y} - y_0} \int_{y_0+B}^{\bar{y}} y_1 - B dy_1$$

$$\begin{aligned} \max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{1}{\bar{y} - y_0} (y_0 + B - y_0)y_0 + \frac{1}{\bar{y} - y_0} \int_{y_0+B}^{\bar{y}} y_1 dy_1 \\ - \frac{1}{\bar{y} - y_0} \int_{y_0+B}^{\bar{y}} B dy_1 \end{aligned}$$

$$\begin{aligned} \max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{1}{\bar{y} - y_0} B y_0 + \frac{1}{\bar{y} - y_0} \frac{\bar{y}^2 - (y_0 + B)^2}{2} \\ - \frac{1}{\bar{y} - y_0} (\bar{y} - y_0 - B)B \end{aligned}$$

$$\begin{aligned} \max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{1}{\bar{y} - y_0} B y_0 + \frac{1}{\bar{y} - y_0} \left(\frac{\bar{y}^2}{2} - \frac{y_0^2 + 2y_0B + B^2}{2} \right) \\ - \frac{1}{\bar{y} - y_0} (\bar{y} - y_0 - B)B \end{aligned}$$

$$\begin{aligned}
& \max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{1}{\bar{y} - y_0}By_0 + \frac{1}{\bar{y} - y_0} \left(\frac{\bar{y}^2}{2} - \frac{y_0^2}{2} - \frac{2y_0B}{2} - \frac{B^2}{2} \right) \\
& \quad - \frac{1}{\bar{y} - y_0}(\bar{y} - y_0)B + \frac{1}{\bar{y} - y_0}B^2 \\
& \max_B y_0 - c + \left(1 - \frac{B}{\bar{y} - y_0}\right)B + \frac{By_0}{\bar{y} - y_0} + E(y_1) - \frac{1}{\bar{y} - y_0} \left(\frac{2y_0B}{2} + \frac{B^2}{2} \right) - B + \frac{1}{\bar{y} - y_0}B^2 \\
& \max_B y_0 - c + B - \frac{B^2}{\bar{y} - y_0} + \frac{By_0}{\bar{y} - y_0} + E(y_1) - \frac{y_0B}{\bar{y} - y_0} - \frac{B^2}{2(\bar{y} - y_0)} - B + \frac{1}{\bar{y} - y_0}B^2 \\
& \max_B y_0 - c + B - \frac{B^2}{\bar{y} - y_0} + E(y_1) - \frac{B^2}{2(\bar{y} - y_0)} - B + \frac{1}{\bar{y} - y_0}B^2 \\
& \max_B y_0 - c - \frac{B^2}{\bar{y} - y_0} + E(y_1) - \frac{B^2}{2(\bar{y} - y_0)} + \frac{B^2}{\bar{y} - y_0} \\
& \max_B y_0 - c + E(y_1) - \frac{B^2}{2(\bar{y} - y_0)} \\
& \max_B - \frac{B^2}{2(\bar{y} - y_0)} + y_0 - c + E(y_1)
\end{aligned}$$

s. t.

$$\frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}} < B < \frac{\bar{y} - y_0}{2} + \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}$$

$$B \leq \bar{y} - y_0$$

For $y_0 = 0$ and $\bar{y} = 1$, this becomes:

$$\max_B - \frac{B^2}{2} - c + \frac{1}{2}$$

s. t.

$$c \leq \frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{2}\sqrt{1-4c} < B < \frac{1}{2} + \frac{1}{2}\sqrt{1-4c}$$

$$B \leq 1$$

Since this is an inverted parabola with no term in B, its symmetry axis is $x = 0$. This means that its maximum is at $B = 0$. Since the government must satisfy the budget constraint, the solution to the maximization problem is the minimum necessary to satisfy the budget constraint, as long as it gives it yields positive expected utility:

$$B \geq \frac{c - y_0}{q} \text{ and } q = 1 - \frac{B}{\bar{y} - y_0}$$

$$\left(1 - \frac{B}{\bar{y} - y_0}\right)B = c - y_0$$

$$B - \frac{B^2}{\bar{y} - y_0} = c - y_0$$

$$\frac{B^2}{\bar{y} - y_0} - B + (c - y_0) = 0$$

$$B = \frac{1 \pm \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}}{\frac{2}{\bar{y} - y_0}}$$

$$B = \frac{\bar{y} - y_0}{2} \pm \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}$$

Taking the minimum of the two:

$$B = \frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}$$

Which is:

$$B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4(c)}$$

The equilibrium price is:

$$q = 1 - \frac{B}{\bar{y} - y_0}$$

$$\text{At } B = \frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}$$

$$q = 1 - \frac{\left(\frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}} \right)}{\bar{y} - y_0}$$

$$q = 1 - \left(\frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}} \right)$$

$$q = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}$$

So the market equilibrium is:

$$\left(q = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}, B = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4c} \right)$$

3. Marginal cost and marginal benefit of borrowing:

To compute marginal cost and marginal benefit, we take, from before:

$$\max_B y_0 - c + B - \frac{B^2}{\bar{y} - y_0} + \frac{1}{\bar{y} - y_0} \left(\frac{\bar{y}^2 - y_0^2}{2} \right) + \frac{1}{\bar{y} - y_0} \left(B(y_0 - \bar{y}) + \frac{B^2}{2} \right)$$

This can be re-written as:

$$\max_B y_0 - c + B - \frac{B^2}{\bar{y} - y_0} + E(y_1) - B + \frac{B^2}{2(\bar{y} - y_0)}$$

$$\max_B y_0 - c + B \left(1 - \frac{B}{\bar{y} - y_0} \right) + E(y_1) - B \left(1 - \frac{B}{2(\bar{y} - y_0)} \right)$$

$$\text{marginal benefit: } 1 - \frac{2B}{\bar{y} - y_0}$$

$$\text{marginal cost: } 1 - \frac{B}{\bar{y} - y_0}$$

4. Market equilibrium with perfect enforcement:

The first-best contract under perfect competition would be the solution to the following constrained optimization problem:

$$\max_B y_0 - c + qB + \int_{y_0}^{\bar{y}} y_1 - B dF(y_1)$$

$$s. t$$

$$E(\pi) = 0$$

$$B \geq \frac{c - y_0}{q}$$

Zero expected profits:

$$E(\pi) = 0$$

$$qB = E(B)$$

$$qB = B$$

$$q = 1$$

So,

$$\max_B y_0 - c + qB + \int_{y_0}^{\bar{y}} y_1 - B dF(y_1)$$

$$s. t$$

$$q = 1$$

$$B \geq c - y_0$$

Inserting the price schedule into the government's problem and solving for B:

$$\max_B y_0 - c + qB + \frac{1}{\bar{y} - y_0} \int_{y_0}^{\bar{y}} y_1 - B d(y_1)$$

$$\max_B y_0 - c + qB + \frac{1}{\bar{y} - y_0} \int_{y_0}^{\bar{y}} (y_1 - B) d(y_1)$$

$$\max_B y_0 - c + qB + \frac{1}{\bar{y} - y_0} \int_{y_0}^{\bar{y}} y_1 d(y_1) - \frac{1}{\bar{y} - y_0} \int_{y_0}^{\bar{y}} B d(y_1)$$

$$\max_B y_0 - c + qB + \frac{1}{\bar{y} - y_0} \frac{\bar{y}^2 - y_0^2}{2} - \frac{1}{\bar{y} - y_0} (\bar{y} - y_0) B$$

$$\max_B y_0 - c + qB + E(y_1) - B$$

$$\max_B y_0 - c + B + E(y_1) - B$$

$$\max_B y_0 - c + E(y_1)$$

s. t

$$B \geq c - y_0$$

So we see that expected utility is independent of the level of debt. Let us suppose that the government would borrow just enough to avoid default in period 0:

$$B = c - y_0$$

For $y_0 = 0$ and $\bar{y} = 1$, equilibrium would be:

$$(B = c, q = 1)$$

And the government's expected payoff would be $\frac{1}{2} - c$.

Moreover, it is individually rational to borrow any amount c instead of only $c < \frac{1}{4}$.

5. Proof of Lemma 1

Lemma 1: *With symmetric information, lending exists in equilibrium for $c < \frac{1}{4}$ for a type 1 government.*

The condition for it to be individually rational to borrow the optimal amount of debt is that the overall expected payoff of the government is greater than 0, since defaulting in period 0 (and therefore not moving on to period 1) yields a payoff of 0. The condition is given below:

$$\text{expected payoff} \geq 0 \text{ at } B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$$

The solution to this is:

$$c \leq \frac{1}{4}$$

It can also be seen through the maximum of the price schedule*B.

The expected payoff at the optimal level of debt is:

$$\begin{aligned}
& -\frac{B^2}{2(\bar{y} - y_0)} + y_0 - c + E(y_1) \\
& \text{At } B = \frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}} \\
& -\frac{\left(\frac{\bar{y} - y_0}{2} - \frac{\bar{y} - y_0}{2} \sqrt{1 - \frac{4(c - y_0)}{\bar{y} - y_0}}\right)^2}{2(\bar{y} - y_0)} + y_0 - c + E(y_1) \geq 0
\end{aligned}$$

For $y_0 = 0$ and $\bar{y} = 1$,

$$\text{expected payoff} \geq 0 \text{ at } B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$$

$$-\frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}\right)^2}{2} - c + \frac{1}{2} \geq 0$$

$$\frac{1}{4} + \frac{1}{4}\sqrt{1 - 4c} - \frac{c}{2} \geq 0$$

$$\frac{1}{2}\left(\frac{1}{2} - c\right) + \frac{1}{4}\sqrt{1 - 4c} > 0$$

As long as $c \leq \frac{1}{4}$, it is optimal to borrow $\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$, since $c < \frac{1}{4}$ ensures that this condition is always satisfied.

6. Proof that the optimal choice of H by a type 2 is a corner solution

The hiding choice of a type 2 government is a corner solution: either 0 or K.

$$\max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0 dF(y_1) + \int_{B+K}^1 (y_1 - (B + K)) dF(y_1)$$

This becomes:

$$q(B + K) - (B + K) + \frac{(B + K)^2}{2} - c + \frac{1}{2}$$

(...)

Take $q = 1 - B - x$

$$qB - B + \frac{B^2}{2} - c + \frac{1}{2} + \frac{H^2}{2} - xH$$

By the time a type 2 government chooses H, q and B are a given, so the first part of the function is a given. The second part is convex in H (decreasing). It reaches a minimum at $x=H$. In order for hiding H to increase the expected payoff of a type 2 government, the payoff from hiding H has to be greater than the payoff of not hiding, that is:

$$qB - B + \frac{B^2}{2} - c + \frac{1}{2} + \frac{H^2}{2} - xH > qB - B + \frac{B^2}{2} - c + \frac{1}{2}$$

$$\frac{H^2}{2} > xH$$

$$\frac{H}{2} > x$$

So, if $x < \frac{H}{2}$, it is optimal to choose the maximum H possible, which is K. However, any other choice of H would not be an equilibrium, since lenders would be making negative expected profits, because they would not be pricing the entire default probability of the choice of H.

If $x > \frac{H}{2}$, it is optimal to choose $H=0$. But this would not be an equilibrium either, because lenders would be making a positive expected profit, since they would be pricing a default risk higher than that corresponding to $H=0$.

7. Price derivation for a type 2 government:

$$E(\pi) = 0$$

$$-qB + E(B) = 0$$

$$qB = E(B)$$

$$qB = B * Prob(y_1 \geq y_0 + B + H) + 0 * Prob(y_1 < y_0 + B)$$

$$q(B) = 1 - \frac{B + H}{\bar{y} - y_0}$$

For $y_0 = 0$ and $y_1 = 1$, this is:

$$q(B) = 1 - (B + H)$$

8. Market equilibrium for a type 2 government:

$$\max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0 dF(y_1) + \int_{B+K}^1 (y_1 - (B + K)) dF(y_1)$$

s. t.

$$q * (B + K) \geq c$$

$$q(B) = 1 - B - K$$

$$B + K < 1$$

Solution:

Since the choice of H is a corner choice (either 0 or H) and since if lenders price H=0 there is no way a type 2 government will not hide K, a necessary condition for a lending equilibrium

(that is, at least B positive) is that lenders price the default risk of K and the government needs to borrow B and K . That is given by B that solves:

$$c = (1 - B - K) * (B + K)$$

Which is

$$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c} - K$$

So a type 2 government borrows

$$B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c} - K$$

$$H = K$$

9. Equilibrium price under asymmetric information

The zero expected profits condition is now:

$$E(\pi) = -qB + P(B(1 - B - K)) + (1 - P)(B(1 - B))$$

$$E(\pi) = 0$$

$$-qB + P(B(1 - B - K)) + (1 - P)(B(1 - B)) = 0$$

$$qB = PB(1 - B - K) + (1 - P)B(1 - B)$$

$$q = P(1 - B - K) + (1 - P)(1 - B)$$

$$q = P - PB - PK + 1 - B - P + PB$$

$$q = -PK + 1 - B$$

$$q = 1 - B - PK$$

10. Market equilibrium under asymmetric information

The equilibrium if the government is of type 1 is the pair (q, B) that solves:

$$\max_B -c + qB + \int_0^B 0dF(y_1) + \int_B^1 (y_1 - B)dF(y_1)$$

$$s. t.$$

$$qB \geq c$$

$$q(B) = 1 - B - PK$$

$$B < 1$$

The equilibrium if the government is of type 2 is the trio (q, B, K) that solves:

$$\max_B y_0 - c + q * (B + K) + \int_0^{B+K} 0dF(y_1) + \int_{B+K}^1 (y_1 - (B + K))dF(y_1)$$

$$s. t.$$

$$qB \geq c$$

$$q(B) = 1 - B - PK$$

$$B + K < 1$$

In a lending equilibrium under asymmetric information, a type 2 government must satisfy its budget constraint entirely through B , because otherwise a type 2 reveals itself, lenders would adapt the price schedule (decrease it) and a type 2 would be worse-off.

The equilibrium amount of borrowing for a type 1 is:

$$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c}$$

The equilibrium amount of borrowing for a type 2 is:

$$B = \frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c}, H = K, \text{ if } P < \frac{1}{2}$$

$$B = 0, H = 0, \text{ if } P > \frac{1}{2}$$

11. Proof of Lemma 3

Lemma 3: *With asymmetric information about the government's type, lending exists in equilibrium for $c < \frac{1}{4} - PK \left(\frac{2-PK}{4} \right)$ for a type 1 and for $c < \frac{1}{4}$ for a type 2, if $P < \frac{1}{2}$. If $P > \frac{1}{2}$, the market is in a zero lending equilibrium for any c for a type 1 and lending exists for $c > \frac{1}{4} - \frac{1}{2}K(1 - \frac{K}{2})$ for a type 2.*

It must be incentive compatible for a type 2 to hide borrowing, since hiding has benefits given that the price is higher than $1-B-K$, but it also has the cost of a higher debt burden in $t = 1$, which increases the default probability and default is costly. A type 2's payoff from choosing $H=K$ is:

$$y_0 - c + q * (B + H) + \int_0^{B+H} 0dF(y_1) + \int_{B+H}^1 (y_1 - (B + H))dF(y_1)$$

While the payoff from choosing $H=0$ is:

$$-c + qB + \int_0^B 0dF(y_1) + \int_B^1 (y_1 - B)dF(y_1)$$

So a type 2 chooses $H=K$ if and only if:

$$\begin{aligned} y_0 - c + q * (B + H) + \int_0^{B+H} 0dF(y_1) + \int_{B+H}^1 (y_1 - (B + H))dF(y_1) \\ > -c + qB + \int_0^B 0dF(y_1) + \int_B^1 (y_1 - B)dF(y_1) \end{aligned}$$

Which yields $P < \frac{1}{2}$.

Maximum of the function qB at $q = 1 - B - PK$:

$$qB = c$$

$$(1 - B - PK)B = c$$

$$B - B^2 - PBK = c$$

$$-B^2 + B(1 - PK) - c = 0$$

Taking the maximizer:

$$\frac{d(-B^2 + B(1 - PK) - c)}{dB} = 0$$

$$-2B + 1 - PK = 0$$

$$\frac{1 - PK}{2} = B$$

Computing the maximum:

$$-B^2 + B(1 - PK) - c = 0$$

$$-\left(\frac{1 - PK}{2}\right)^2 + \left(\frac{1 - PK}{2}\right)(1 - PK) - c = 0$$

Yields $c = \frac{1}{4} - PK \left(\frac{2 - PK}{4}\right)$.

If $c < \frac{1}{4} - PK \left(\frac{2 - PK}{4}\right)$, borrowing is IR for a type 1. If $c > \frac{1}{4} - PK \left(\frac{2 - PK}{4}\right)$, then a type 2 government is unable to raise enough financing to cover c only through B , which implies that a type 2 reveals itself.

12. Proof of Proposition 2

Proposition 2:

- If $P < \frac{1}{2}$, asymmetric information about the government's type induces a transfer of welfare from a type 1 government to a type 2. In this case, asymmetric information decreases the welfare of a type 1 and increases that of a type 2 only if P or c are sufficiently small.
- If $P > \frac{1}{2}$, asymmetric information decreases the welfare of a type 1 by ceasing to provide it with market lending and decreases the welfare of a type 2 by ceasing to provide it with market lending for low enough financing needs.
- Under asymmetric information, when $P < \frac{1}{2}$, a type 1's welfare is decreasing in P and K , but a type 2's welfare is decreasing in P and increasing in K if P is sufficiently low.

Derivative of maximum c for a type 1 with respect to P and K

$$\frac{d \frac{1}{4} - \frac{PK(2 - PK)}{4}}{dP} < 0$$

$$\frac{d \frac{1}{4} - \frac{PK(2 - PK)}{4}}{dK} < 0$$

Derivative of type 1's expected payoff with respect to P and K :

$$\frac{d \frac{1}{4} + \frac{1}{4}(1 + PK)\sqrt{(1 - PK)^2 - 4c} - \frac{c}{2} + \frac{P^2 K^2}{4}}{dP} < 0$$

$$\frac{d \frac{1}{4} + \frac{1}{4}(1 + PK)\sqrt{(1 - PK)^2 - 4c} - \frac{c}{2} + \frac{P^2 K^2}{4}}{dK} < 0$$

If $P < 1/2$.

13. Proof of Lemma 4

Equilibrium 1 (inspect, not hide):

Notice, first, that the default probability of the government is not affected by the existence of a hiding penalty, since the hiding penalty does not affect the rule for optimal default:

Assuming inspection, a type 2 that hides optimally defaults iff:

$$y_1 - B - K - x < y_0 - x$$

That is,

$$y_1 - B - K < y_0$$

Which is the same rule on the income realization as without the penalty.

Assuming inspection, the expected payoff from hiding is the following:

$$\begin{aligned}
 & -c + q(B + K) + \int_0^{B+K} (0 - x)dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x]dF(y_1) \\
 & -c + q(B + K) - \int_0^{B+K} (x)dF(y_1) + \int_{B+K}^1 (y_1)dF(y_1) - \int_{B+K}^1 (B + K + x)dF(y_1) \\
 & -c + q(B + K) - (B - 0)x - (B + K - B)x + \frac{(1^2 - (B + K)^2)}{2} \\
 & - (1 - (B + K))(B + K + x) \\
 & -c + q(B + K) - Bx - Kx + \frac{(1^2 - B^2 - 2BK - K^2)}{2} - (B + K + x) \\
 & + (B + K)(B + K + x) \\
 & -c + q(B + K) - (B + K)x + \frac{1}{2} - \frac{B^2}{2} - BK - \frac{K^2}{2} - (B + K + x) + (B + K)^2 + (B + K)x
 \end{aligned}$$

$$\begin{aligned}
& -c + q(B + K) - (B + K)x + \frac{1}{2} - \frac{B^2}{2} - BK - \frac{K^2}{2} - (B + K + x) + B^2 + 2BK + K^2 \\
& + (B + K)x
\end{aligned}$$

$$-c + q(B + K) + \frac{1}{2} + \frac{B^2}{2} + BK + \frac{K^2}{2} - (B + K + x)$$

Expected payoff from not hiding:

$$-c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

For this equilibrium to hold, the penalty must deter hiding. The type 2 government optimally does not hide iff:

$$-c + q(B + K) + \frac{1}{2} + \frac{B^2}{2} + BK + \frac{K^2}{2} - (B + K + x) < -c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

$$qK + BK + \frac{K^2}{2} - (K + x) < 0$$

$$qK + BK + \frac{K^2}{2} - K - x < 0$$

$$qK + BK + \frac{K^2}{2} - K < x$$

$$x > K \left(q + B + \frac{K}{2} - 1 \right)$$

Or, alternatively:

$$qK < -BK - \frac{K^2}{2} + (K + x) < 0$$

$$qK < \left(1 - B - \frac{K}{2} \right) K + x$$

That is, if the benefit of hiding (the price) is lower than the cost of hiding (the penalty plus the cost of repaying K or defaulting on K (which is to have GDP equal to 0)).

Assuming that a type 2 government does not hide, the zero profits price is:

$$-qB + B(1 - B) - M = 0$$

$$qB = B(1 - B) - M$$

$$q = 1 - B - \frac{M}{B}$$

Inserting the price schedule, the necessary penalty is therefore:

$$\left(1 - B - \frac{M}{B}\right)K < \left(1 - B - \frac{K}{2}\right)K + x$$

Part of the benefit is equal to the cost. Eliminating those:

$$\left(-\frac{M}{B}\right)K < \left(-\frac{K}{2}\right)K + x$$

Re-writing, the minimum deterring penalty under commitment to inspect is:

$$x > K\left(\frac{K}{2} - \frac{M}{B}\right)$$

Solving for B and then replacing B for its optimal expression, we get:

The price in this equilibrium is:

$$q = 1 - B - \frac{M}{B}$$

The expression for optimal B is:

$$\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)}$$

So we get:

$$\frac{x}{K} > \frac{K}{2} - \frac{M}{B}$$

$$\frac{x}{K} - \frac{K}{2} > -\frac{M}{B}$$

$$\frac{M}{B} > -\frac{x}{K} + \frac{K}{2}$$

$$\frac{M}{B} > \frac{K^2 - 2x}{2K}$$

$$2MK > B(K^2 - 2x)$$

$$2MK > \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)}\right)(K^2 - 2x)$$

If $x > \frac{K^2}{2}$, this is always satisfied (because $K^2 - 2x < 0$)

If $x < \frac{K^2}{2}$:

$$2MK > \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)}\right)(K^2 - 2x)$$

$$2MK > \frac{1}{2}(K^2 - 2x) - \frac{1}{2}\sqrt{1 - 4(M + c)}(K^2 - 2x)$$

$$\frac{1}{2}\sqrt{1 - 4(M + c)}(K^2 - 2x) > \frac{1}{2}(K^2 - 2x) - 2MK$$

$$\sqrt{1 - 4(M + c)}(K^2 - 2x) > (K^2 - 2x) - 4MK$$

$$\sqrt{1 - 4(M + c)} > \frac{(K^2 - 2x) - 4MK}{(K^2 - 2x)}$$

If $(K^2 - 2x) - 4MK < 0$, this is always satisfied. Re-writing:

$$(K^2 - 2x) < 4MK$$

$$\frac{K^2 - 2x}{4K} < M$$

$$M > -\frac{2x}{4K} + \frac{K^2}{4K}$$

$$M > -\frac{x}{2K} + \frac{K}{4}$$

If $(K^2 - 2x) - 4MK > 0$, then:

$$1 - 4(M + c) > \left(\frac{(K^2 - 2x) - 4MK}{(K^2 - 2x)}\right)^2$$

$$1 - \left(\frac{(K^2 - 2x) - 4MK}{(K^2 - 2x)}\right)^2 > 4(M + c)$$

$$1 - 4M - \left(\frac{(K^2 - 2x) - 4MK}{(K^2 - 2x)}\right)^2 > 4c$$

$$\frac{1}{4} - M - \frac{\left(1 - \frac{4MK}{(K^2 - 2x)}\right)^2}{4} > c$$

$$c < \frac{1}{4} - M - \frac{1}{4}\left(1 - \frac{4MK}{(K^2 - 2x)}\right)^2$$

$$c < \frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{4MK}{2x - K^2}\right)^2$$

So, when $M < -\frac{x}{2K} + \frac{K}{4}$, there is another upper bound on c , which is:

$$c < \frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{4MK}{2x - K^2}\right)^2$$

The price schedule imposes an upper bound on c , given by the maximum of this (concave) function:

$$qB$$

$$(1 - B - \frac{M}{B})B$$

$$B - B^2 - M$$

$$\frac{d}{dB} = 0$$

$$1 - 2B = 0$$

$$B = \frac{1}{2}$$

qB at $\frac{1}{2}$ is:

$$(1 - \frac{1}{2} - 2M)\frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{4} - M$$

$$\frac{1}{4} - M$$

So there is an upper bound on c , which is:

$$c < \frac{1}{4} - M$$

So, commitment restores lending, for:

- $c < \frac{1}{4} - M$, if $M > -\frac{x}{2K} + \frac{K}{4}$

- $c < \frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{4MK}{2x-K^2}\right)^2$, if $M < -\frac{x}{2K} + \frac{K}{4}$

The subset of parameters where $\frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{4MK}{2x-K^2}\right)^2$ is positive is:

$$\frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{4MK}{2x-K^2}\right)^2 > 0$$

$$\frac{1}{4} - M - \frac{1}{4}\left(1 + \frac{8MK}{2x-K^2} + \frac{16M^2K^2}{(2x-K^2)^2}\right) > 0$$

$$-M - \frac{1}{4}\left(\frac{8MK}{2x-K^2} + \frac{16M^2K^2}{(2x-K^2)^2}\right) > 0$$

$$-M - \frac{2MK}{2x-K^2} - \frac{4M^2K^2}{(2x-K^2)^2} > 0$$

$$M + \frac{2MK}{2x-K^2} + \frac{4M^2K^2}{(2x-K^2)^2} < 0$$

$$\frac{M(2x-K^2)^2 + 2MK(2x-K^2) + 4M^2K^2}{(2x-K^2)^2} < 0$$

$$M(2x-K^2)^2 + 2MK(2x-K^2) + 4M^2K^2 < 0$$

$$M(2K(2x-K^2) + (2x-K^2)^2) + 4M^2K^2 < 0$$

$$(2K(2x-K^2) + (2x-K^2)^2) + 4MK^2 < 0$$

$$4MK^2 < -2K(2x-K^2) - (2x-K^2)^2$$

$$4MK^2 < (2x-K^2)(-2K - (2x-K^2))$$

$$4MK^2 < (2x-K^2)(-2K - 2x + K^2)$$

$$M < \frac{(2x-K^2)(-2K - 2x + K^2)}{4K^2}$$

This is always satisfied in the relevant parameter region, that is, the function $\frac{(2x-K^2)(-2K-2x+K^2)}{4K^2}$ necessarily above the function $-\frac{x}{2K} + \frac{K}{4}$. This is so, since the value that the functions take when they cross the horizontal axis is the same, both functions are concave and the value that the function $\frac{(2x-K^2)(-2K-2x+K^2)}{4K^2}$ takes when it crosses the vertical axis is higher than the one that the function $-\frac{x}{2K} + \frac{K}{4}$ takes.

$$\frac{(2x - K^2)(-2K - 2x + K^2)}{4K^2} > -\frac{x}{2K} + \frac{K}{4}$$

First derivative of $\frac{(2x-K^2)(-2K-2x+K^2)}{4K^2}$ wrt x:

$$\frac{2(-2K - 2x + K^2) + (2x - K^2)(-2)}{4K^2} =$$

$$\frac{2(-2K - 2x + K^2) - 2(2x - K^2)}{4K^2} =$$

$$\frac{4K^2 - 8x - 4K}{4K^2} =$$

$$1 - \frac{8x + 4K}{4K^2}$$

$$1 - \frac{8x}{4K^2} - \frac{1}{K}$$

Second derivative wrt x:

$$-\frac{8}{4K^2} =$$

$$-\frac{2}{K^2} < 0$$

Since the second derivative is negative, the function is concave.

The value of the function at $M = 0$ is $\frac{K^2}{2}$, which is the same value that the function $-\frac{x}{2K} + \frac{K}{4}$ takes at $M = 0$. The value that the function takes at $x = 0$ is:

$$\frac{(0 - K^2)(-2K - 0 + K^2)}{4K^2} =$$

$$\frac{(-K^2)(-2K + K^2)}{4K^2} =$$

$$\frac{2K^3 - K^4}{4K^2} =$$

$$\frac{2K^3}{4K^2} - \frac{K^4}{4K^2} =$$

$$\frac{2K}{4} - \frac{K^2}{4} =$$

$$\frac{K}{2} - \frac{K^2}{4} =$$

$$K\left(\frac{1}{2} - \frac{K}{4}\right) =$$

$$\frac{K}{4}\left(\frac{4}{2} - K\right) =$$

$$\frac{K}{4}(2 - K)$$

And this is higher than the value that the function $-\frac{x}{2K} + \frac{K}{4}$ takes at $x = 0$, which is:

$$-\frac{0}{2K} + \frac{K}{4} =$$

$$\frac{K}{4}$$

Because $K < 1$, by assumption.

Q.E.D.

Expected payoff of the government under a commitment equilibrium:

$$\begin{aligned}
 q * (B) - (B) + \frac{(B)^2}{2} - c + \frac{1}{2} = \\
 -(B) + \frac{(B)^2}{2} + \frac{1}{2} \\
 B = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)} \\
 -\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)}\right) + \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4(M + c)}\right)^2}{2} + \frac{1}{2} \\
 -\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4(M + c)} + \frac{\frac{1}{4} - \frac{1}{2}\sqrt{1 - 4(M + c)} + \frac{1}{4}(1 - 4(M + c))}{2} + \frac{1}{2} \\
 \frac{1}{2}\sqrt{1 - 4(M + c)} + \frac{-\frac{1}{2}\sqrt{1 - 4(M + c)} + \frac{1}{4}(-4(M + c))}{2} + \frac{1}{4} \\
 \frac{1}{2}\sqrt{1 - 4(M + c)} + \frac{-\frac{1}{2}\sqrt{1 - 4(M + c)} - (M + c)}{2} + \frac{1}{4} \\
 \frac{1}{2}\sqrt{1 - 4(M + c)} - \frac{1}{4}\sqrt{1 - 4(M + c)} - \frac{(M + c)}{2} + \frac{1}{4} \\
 \frac{1}{4}\sqrt{1 - 4(M + c)} - \frac{M}{2} + \frac{1}{4} - \frac{c}{2} \\
 \frac{1}{4} + \frac{1}{4}\sqrt{1 - 4(M + c)} - \frac{c}{2} - \frac{M}{2}
 \end{aligned}$$

Equilibrium 2: (inspect, hide):

Assuming inspection, a type 2 government optimally hides iff:

$$-c + q(B + K) + \frac{1}{2} + \frac{B^2}{2} + BK + \frac{K^2}{2} - (B + K + x) > -c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

$$qK + BK + \frac{K^2}{2} - (K + x) > 0$$

$$qK + BK + \frac{K^2}{2} - K - x > 0$$

$$qK + BK + \frac{K^2}{2} - K > x$$

$$x < K \left(q + B + \frac{K}{2} - 1 \right)$$

Or, alternatively:

$$qK > -BK - \frac{K^2}{2} + (K + x) < 0$$

$$qK > \left(1 - B - \frac{K}{2} \right) K + x$$

That is, if the benefit of hiding (the price) is higher than the cost of hiding (the penalty plus the cost of repaying K or defaulting on K (which is to have GDP equal to 0)).

Assuming that a type 2 government hides, the zero profits price is:

$$-qB + (1 - P)[B(1 - B) - M] + P[B(1 - B - K) - M + x] = 0$$

$$q(B) = 1 - B - PK - \frac{M}{B} + \frac{Px}{B}$$

Inserting the price schedule, the necessary penalty not to hide is therefore:

$$\left(1 - B - PK - \frac{M}{B} + \frac{Px}{B} \right) K > \left(1 - B - \frac{K}{2} \right) K + x$$

Part of the benefit is equal to the cost. Eliminating those:

$$\left(-PK - \frac{M}{B} + \frac{Px}{B}\right)K > \left(-\frac{K}{2}\right)K + x$$

Re-writing, the maximum penalty in this equilibrium is:

$$x - K \frac{Px}{B} < K \left(\frac{K}{2} - \frac{M}{B} - PK\right)$$

$$x \left(1 - \frac{PK}{B}\right) < K \left(\frac{K}{2} - \frac{M}{B} - PK\right)$$

$$x \left(\frac{B - PK}{B}\right) < K \left(\frac{BK - 2M - 2BPK}{2B}\right)$$

$$x < K \left(\frac{BK - 2M - 2BPK}{2(B - PK)}\right)$$

Re-writing isolating B:

$$2(B - PK)x < K(BK - 2M - 2BPK)$$

$$2Bx - K^2B + 2BPK^2 < -2KM + 2PKx$$

$$B(2x - K^2 + 2PK^2) < 2PKx - 2KM$$

$$B < \frac{2PKx - 2KM}{2x - K^2 + 2PK^2}$$

Numerator:

$$2PKx - 2KM > 0$$

$$2PKx > 2MK$$

$$Px > M$$

Denominator:

$$2x - K^2 + 2PK^2 > 0$$

$$2x > K^2 - 2PK^2$$

$$x > K^2 \frac{1 - 2P}{2}$$

If $P > \frac{1}{2}$, this always holds.

So, if $P > \frac{1}{2}$, for this threshold level of B to be positive (and therefore for this equilibrium to exist), $Px > M$ must hold.

Solving for B and then replacing B for its optimal expression, we get:

The price in this equilibrium is:

$$q = 1 - B - PK - \frac{M}{B} + \frac{Px}{B}$$

The expression for optimal B is:

$$\frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)}$$

So we get:

$$\frac{1}{2}(1 - PK) - \frac{1}{2}\sqrt{(1 - PK)^2 - 4c + 4(Px - M)} < \frac{2PKx - 2KM}{2x - K^2 + 2PK^2}$$

(...)

$$c < \frac{1}{4} - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + (Px - M)$$

$$c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$$

So, if $P > \frac{1}{2}$, the equilibrium in which lenders inspect and a type 2 hides exists for:

- $c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ if $Px > M$
- Does not exist elsewhere

If $P < \frac{1}{2}$, the equilibrium in which lenders inspect and a type 2 hides exists for:

- $c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ if $Px > M$ and $x > \frac{K^2}{2}(1 - 2P)$ or if $Px < M$ and $x < \frac{K^2}{2}(1 - 2P)$
- Does not exist elsewhere

$c < \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px$ is necessarily IR for a type 1, since it is a tighter ceiling than the one imposed by the prevailing price schedule (which is the maximum value that c can take in order for borrowing to be IR for a type 1). This is proven by contradiction: since B is an equal function of c for both government types, the payoff of not hiding has a higher derivative (in absolute terms) than the payoff of hiding, otherwise the payoff of hiding would not be higher than the payoff of not hiding. If there were some c' such that $qB < 0$ but the payoff of hiding was higher than the payoff of not hiding, then the payoff of hiding would be higher than the payoff of not hiding for every $c > c'$. Therefore, the upper bound on c imposed by the requirement that hiding is IC for a type 2 must be lower than the upper bound imposed by the requirement that $qB > c$ at the prevailing market price, which is the requirement that makes borrowing IR for a type 1.

14. Proof of Lemma 5

Equilibrium 1:

Expected payoff under hiding:

$$q(B + K) + \int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1)$$

$$q(B + K) + 0 - \int_B^{B+K} x dF(y_1) + \int_{B+K}^1 y_1 dF(y_1) - \int_{B+K}^1 (B + K) dF(y_1)$$

$$q(B + K) - (B + K - B)x + \left(\frac{1^2 - (B + K)^2}{2} \right) - (1 - (B + K))(B + K)$$

$$q(B + K) - Kx + \left(\frac{1^2 - B^2 - 2BK - H^2}{2} \right) - (B + K) + (B + K)^2$$

$$q(B + K) - Kx + \frac{1}{2} - \frac{B^2}{2} - BK - \frac{K^2}{2} - (B + K) + B^2 + 2BK + K^2$$

$$q(B + K) - Kx + \frac{1}{2} + \frac{B^2}{2} + BK + \frac{K^2}{2} - (B + K)$$

Expected payoff from not hiding:

$$qB + \frac{1}{2} + \frac{B^2}{2} - B$$

Assuming that lenders inspect if default is perfectly informative, the government hides if and only if the difference in expected payoffs is negative:

$$qK - Kx + BK + \frac{K^2}{2} - K < 0$$

$$qK + BK + \frac{K^2}{2} - K < Kx$$

$$x > \frac{K \left(q + B + \frac{K}{2} - 1 \right)}{K}$$

$$x > q + B + \frac{K}{2} - 1$$

The price that yields zero expected profits when the government does not hide (and given the rule for inspecting, lenders do not inspect), is:

$$q = 1 - B$$

So the penalty becomes:

$$x > 1 - B + B + \frac{K}{2} - 1$$

$$x > \frac{K}{2}$$

The expected payoff of the government is:

$$-c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

Plugging in the zero expected profits price

$$q = 1 - B$$

The expected payoff of the government becomes:

$$-c + (1 - B)B + \frac{1}{2} + \frac{B^2}{2} - B =$$

$$-c + B - B^2 + \frac{1}{2} + \frac{B^2}{2} - B =$$

$$-\frac{B^2}{2} + \frac{1}{2} - c$$

Conditions on parameters of Equilibrium 1:

When the penalty is high, the first-best price is implemented by the mechanism. Lenders do not inspect, in equilibrium, and a type 2 government does not hide.

The price is:

$$q = 1 - B$$

Optimal B is:

$$\frac{1}{2} - \frac{1}{2}\sqrt{1 - 4c}$$

The price schedule imposes an upper bound on c, given by the maximum of this (concave) function:

$$qB$$

$$(1 - B)B$$

$$B - B^2$$

$$\frac{d}{dB} = 0$$

$$1 - 2B = 0$$

$$B = \frac{1}{2}$$

qB at $\frac{1}{2}$ is:

$$\left(1 - \frac{1}{2}\right)\frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{4}$$

$$\frac{1}{4}$$

So there is an upper bound on c , which is:

$$c < \frac{1}{4}$$

Second, the price cannot be negative. It never is.

Third, the penalty has to be effective in deterring hiding. This does not impose any constraint on c , since the requisite for an effective penalty is:

$$x > \frac{K}{2}$$

Fourth, lenders must have incentives to inspect, so:

$$x > M$$

Equilibrium 3:

When

$$M < Px < x$$

Assuming that lenders inspect when it pays off and assuming a mixed strategy for a type 2 government, the government's payoff from hiding is the following:

$$\begin{aligned}
& -c + q(B + K) + \gamma \left(\int_0^{B+K} (0 - x) dF(y_1) + \int_{B+K}^1 [y_1 - (B + K) - x] dF(y_1) \right) \\
& + (1 - \gamma) \left(\int_0^B (0) dF(y_1) + \int_B^{B+K} (0 - x) dF(y_1) \right. \\
& \left. + \int_{B+K}^1 [y_1 - (B + K)] dF(y_1) \right) \\
& -c + q(B + K) + \gamma \left((B + K)(-x) + \frac{1^2 - (B + K)^2}{2} - (1 - (B + K)(B + K + x)) \right) \\
& + (1 - \gamma) \left(K(-x) + \frac{1^2 - (B + K)^2}{2} - (1 - (B + K))(B + K) \right) \\
& -c + q(B + K) + \gamma \left(-x(B + K) + \frac{1}{2} - \frac{(B + K)^2}{2} - (B + K + x) + (B + K)(B + K + x) \right) \\
& + (1 - \gamma) \left(-xK + \frac{1}{2} - \frac{(B + K)^2}{2} - (B + K) + (B + K)^2 \right) \\
& -c + q(B + K) + \frac{1}{2} + \gamma \left(-x(B + K) - \frac{(B + K)^2}{2} - (B + K + x) + (B + K)x + (B + K)^2 \right) \\
& + (1 - \gamma) \left(-xK - \frac{(B + K)^2}{2} - (B + K) + (B + K)^2 \right) \\
& -c + q(B + K) + \frac{1}{2} + \gamma \left(\frac{(B + K)^2}{2} - (B + K + x) \right) \\
& + (1 - \gamma) \left(-xK + \frac{(B + K)^2}{2} - (B + K) \right) \\
& -c + q(B + K) + \frac{1}{2} + \frac{(B + K)^2}{2} - (B + K) - x\gamma + (1 - \gamma)(-xK)
\end{aligned}$$

Expected payoff of not hiding:

$$-c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

Mix iff:

$$-c + q(B + K) + \frac{1}{2} + \frac{(B + K)^2}{2} - (B + K) - x\gamma + -xK(1 - \gamma) = -c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

$$qK + BK + \frac{K^2}{2} - K - x\gamma + -xK(1 - \gamma) = 0$$

$$qK + BK + \frac{K^2}{2} - K = x\gamma + xK(1 - \gamma)$$

$$K \left(q + B + \frac{K}{2} - 1 \right) = x\gamma + xK - xK\gamma$$

$$K \left(q + B + \frac{K}{2} - 1 \right) - xK = \gamma(x - xK)$$

$$K \left(q + B + \frac{K}{2} - 1 \right) - xK = \gamma x(1 - K)$$

$$\gamma = \frac{K \left(q + B + \frac{K}{2} - 1 \right) - xK}{x(1 - K)}$$

$$\gamma = \frac{K \left(q + B + \frac{K}{2} - 1 - x \right)}{x(1 - K)}$$

OR, alternatively:

Expected payoff of hiding:

$$\begin{aligned}
& K \left(-c + q(B + K) + \int_B^{B+K} (0 - x) dF(y_1 \setminus B < y_1 < B + K) \right) \\
& + B \left(\gamma \left(-c + q(B + K) + \int_0^B (0 - x) dF(y_1 \setminus y_1 < B) \right) \right. \\
& + (1 - \gamma) \left(-c + q(B + K) + \int_0^B (0) dF(y_1 \setminus y_1 < B) \right) \Bigg) \\
& + (1 \\
& - (B + K)) \left(\gamma \left(-c + q(B + K) \right. \right. \\
& + \int_{B+K}^1 (y_1 - (B + K) - x) dF(y_1 \setminus B + K < y_1 < 1) \Bigg) \\
& + (1 - \gamma) \left(-c + q(B + K) \right. \\
& + \left. \left. \int_{B+K}^1 (y_1 - (B + K)) dF(y_1 \setminus B + K < y_1 < 1) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& K \left(-c + q(B + K) + \frac{1}{K}((B + K - B)(-x)) \right) \\
& + B \left(\gamma \left(-c + q(B + K) + \frac{1}{B}(B - 0)(-x) \right) + (1 - \gamma)(-c + q(B + K)) \right) \\
& + (1 \\
& - (B + K)) \left(\gamma \left(-c + q(B + K) + \frac{1}{1 - (B + K)} \left(\frac{1^2 - (B + K)^2}{2} \right) \right. \right. \\
& \left. \left. - \frac{1}{1 - (B + K)} (1 - (B + K))(B + K + x) \right) \right. \\
& \left. + (1 - \gamma) \left(-c + q(B + K) + \frac{1}{1 - (B + K)} \left(\frac{1^2 - (B + K)^2}{2} \right) \right. \right. \\
& \left. \left. - \frac{1}{1 - (B + K)} (1 - (B + K))(B + K) \right) \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& K(-c + q(B + K) - x) + B \left(\gamma(-c + q(B + K) - x) + (1 - \gamma)(-c + q(B + K)) \right) \\
& + (1 - (B + K)) \left(\gamma \left(-c + q(B + K) + \frac{1 + B + K}{2} - (B + K + x) \right) \right. \\
& \left. + (1 - \gamma) \left(-c + q(B + K) + \frac{1 + B + K}{2} - (B + K) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& K(-c + q(B + K) - x) + B(-c + q(B + K) - \gamma x) \\
& + (1 - (B + K)) \left(-c + q(B + K) + \frac{1 + B + K}{2} - (B + K) - \gamma x \right) \\
& -c + q(B + K) - Kx - B\gamma x + (1 - (B + K)) \left(\frac{1 + B + K}{2} - (B + K) - \gamma x \right) \\
& -c + q(B + K) - Kx - B\gamma x + (1 - (B + K)) \left(\frac{1 - B - K}{2} - \gamma x \right)
\end{aligned}$$

$$-c + q(B + K) - Kx - B\gamma x + \frac{1 - B - K}{2} - \gamma x - (B + K) \left(\frac{1 - B - K}{2} \right) + (B + K)\gamma x$$

$$-c + q(B + K) - Kx - B\gamma x + \frac{1}{2} - \frac{B}{2} - \frac{K}{2} - \gamma x - \frac{(B + K)}{2} + \frac{(B + K)^2}{2} + B\gamma x + K\gamma x$$

$$-c + q(B + K) - Kx + \frac{1}{2} - \frac{B}{2} - \frac{K}{2} - \gamma x - \frac{B}{2} - \frac{K}{2} + \frac{(B + K)^2}{2} + K\gamma x$$

$$-c + q(B + K) - Kx + \frac{1}{2} - B - K - \gamma x + \frac{(B + K)^2}{2} + K\gamma x$$

$$-c + q(B + K) + \frac{1}{2} - (B + K) - Kx - \gamma x + \frac{(B + K)^2}{2} + K\gamma x$$

$$-c + q(B + K) + \frac{(B + K)^2}{2} + \frac{1}{2} - (B + K) - Kx - \gamma(x - Kx)$$

$$-c + q(B + K) + \frac{(B + K)^2}{2} + \frac{1}{2} - (B + K) - Kx - \gamma x(1 - K)$$

Expected payoff from not hiding:

$$-c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

Mix iff:

$$-c + q(B + K) + \frac{(B + K)^2}{2} + \frac{1}{2} - (B + K) - Kx - \gamma x(1 - K) = -c + qB + \frac{1}{2} + \frac{B^2}{2} - B$$

$$qK + BK + \frac{K^2}{2} - K - Kx - \gamma x(1 - K) = 0$$

$$qK + BK + \frac{K^2}{2} - K - Kx = \gamma x(1 - K)$$

$$K \left(q + B + \frac{K}{2} - 1 - x \right) = \gamma x(1 - K)$$

$$\gamma = \frac{K \left(q + B + \frac{K}{2} - 1 - x \right)}{x(1-K)}$$

Expected payoff from inspecting:

$$\begin{aligned} & -qB + P[\rho(B(1-B-K) + x - M) + (1-\rho)(B(1-B) - (1-K)M)] \\ & \quad + (1-P)[B(1-B) - (1-K)M] \\ & = -qB + P[B(1-B) + \rho(-BK + x - M) + (1-\rho)(-(1-K)M)] \\ & \quad + (1-P)[B(1-B) - (1-K)M] \\ & = -qB + B(1-B) + P\rho(-BK + x - M) + P(1-\rho)(-(1-K)M) \\ & \quad + (1-P)[-(1-K)M] \\ & = -qB + B(1-B) + P\rho(-BK + x - M) - P(1-K)M + P\rho(1-K)M \\ & \quad - (1-P)(1-K)M \end{aligned}$$

Expected profit from partial inspection:

$$\begin{aligned} & -qB + P[\rho(B(1-B-K) + K(x-M)) + (1-\rho)(B(1-B))] + (1-P)[B(1-B)] \\ & = -qB + P[B(1-B) + \rho(-BK + K(x-M))] + (1-P)B(1-B) \\ & = -qB + B(1-B) + P\rho(-BK + K(x-M)) \end{aligned}$$

Mix iff:

$$\begin{aligned} & = -qB + B(1-B) + P\rho(-BK + x - M) - P(1-K)M + P\rho(1-K)M \\ & \quad - (1-P)(1-K)M = -qB + B(1-B) + P\rho(-BK + K(x-M)) \\ & = P\rho(-BK + x - M) - P(1-K)M + P\rho(1-K)M - (1-P)(1-K)M \\ & = P\rho(-BK + K(x-M)) \end{aligned}$$

$$= P\rho(x - M) - P(1 - K)M + P\rho(1 - K)M - (1 - P)(1 - K)M = P\rho K(x - M)$$

$$= P\rho(x - M) - P(1 - K)M + P\rho M - P\rho KM - (1 - K)M + P(1 - K)M - P\rho Kx \\ + P\rho KM = 0$$

$$= P\rho(x - M) + P\rho M - (1 - K)M - P\rho Kx = 0$$

$$= P\rho x - P\rho M + P\rho M - (1 - K)M - P\rho Kx = 0$$

$$P\rho x - (1 - K)M - P\rho Kx = 0$$

$$P\rho x(1 - K) = (1 - K)M$$

$$\rho = \frac{(1 - K)M}{Px(1 - K)}$$

$$\rho = \frac{M}{Px}$$

Price:

$$-qB + K[P\rho(B(1 - B - K) + x - M) + (1 - P\rho)(B(1 - B))] \\ + B[\gamma(P\rho(B(1 - B - K) + x - M) + (1 - P\rho)(B(1 - B) - M)) \\ + (1 - \gamma)(P\rho(B(1 - B - K)) + (1 - P\rho)(B(1 - B)))] \\ + (1 - (B + K))[\gamma(P\rho(B(1 - B - K) + x - M) + (1 - P\rho)(B(1 - B) - M)) \\ + (1 - \gamma)(P\rho(B(1 - B - K)) + (1 - P\rho)(B(1 - B)))] = 0$$

$$qB = K[P\rho(B(1 - B - K) + x - M) + (1 - P\rho)(B(1 - B))] \\ + (1 - K)[\gamma(P\rho(B(1 - B - K) + x - M) + (1 - P\rho)(B(1 - B) - M)) \\ + (1 - \gamma)(P\rho(B(1 - B - K)) + (1 - P\rho)(B(1 - B)))]$$

$$\begin{aligned}
qB &= K[B(1 - B) + P\rho(-BK + x - M)] \\
&\quad + (1 - K)[\gamma(B(1 - B) - M + P\rho(-BK + x))] \\
&\quad + (1 - \gamma)(B(1 - B) + P\rho(-BK))
\end{aligned}$$

$$qB = K[B(1 - B) - P\rho BK + P\rho(x - M)] + (1 - K)[B(1 - B) - P\rho BK + \gamma(P\rho x - M)]$$

$$qB = B(1 - B) - P\rho BK + K(P\rho(x - M)) + (1 - K)\gamma(P\rho x - M)$$

$$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M) + \frac{(1 - K)}{B}\gamma(P\rho x - M)$$

$$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$$

Inserting the price in gamma:

$$\gamma = \frac{K\left(q + B + \frac{K}{2} - 1 - x\right)}{x(1 - K)}$$

$$\gamma = \frac{K\left(1 - B - P\rho K + \frac{K}{B}P\rho(x - M) + \frac{(1 - K)}{B}\gamma(P\rho x - M) + B + \frac{K}{2} - 1 - x\right)}{x(1 - K)}$$

$$\gamma = \frac{K\left(-P\rho K + \frac{K}{B}P\rho(x - M) + \frac{(1 - K)}{B}\gamma(P\rho x - M) + \frac{K}{2} - x\right)}{x(1 - K)}$$

$$\gamma x(1 - K) = K\left(-P\rho K + \frac{K}{B}P\rho(x - M) + \frac{(1 - K)}{B}\gamma(P\rho x - M) + \frac{K}{2} - x\right)$$

$$\gamma x(1 - K) - K\frac{(1 - K)}{B}\gamma(P\rho x - M) = K\left(-P\rho K + \frac{K}{B}P\rho(x - M) + \frac{K}{2} - x\right)$$

$$\gamma\left(x(1 - K) - K\frac{(1 - K)}{B}(P\rho x - M)\right) = K\left(-P\rho K + \frac{K}{B}P\rho(x - M) + \frac{K}{2} - x\right)$$

$$\gamma = \frac{K \left(-P\rho K + \frac{K}{B} P\rho(x - M) + \frac{K}{2} - x \right)}{x(1 - K) - \frac{K(1 - K)}{B} (P\rho x - M)}$$

In equilibrium,

$$P\rho x - M = 0$$

So

$$\gamma = \frac{K \left(-P\rho K + \frac{K}{B} P\rho(x - M) + \frac{K}{2} - x \right)}{x(1 - K)}$$

$$\gamma = \frac{1}{x(1 - K)} K \left(-P\rho K + \frac{K}{B} P\rho(x - M) + \frac{K}{2} - x \right)$$

Conditions on parameters of Equilibrium 3:

$$q = 1 - B - P\rho K + \frac{K}{B} P\rho(x - M)$$

Substituting rho for its equilibrium expression, the price becomes:

$$q = 1 - B - \frac{M}{x} K + \frac{K}{B} \frac{M}{x} (x - M)$$

The expression of optimal B is:

$$\frac{1}{2x} (x - MK) - \frac{1}{2x} \sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

The upper bound on c that results from the price schedule (the requisite that c must be such that there is a positive amount of bonds (B) that can raise c at the prevailing market price) is the maximum of the function qB . It is the following:

$$c < \frac{1}{4} - MK \left(\frac{2x - MK - 4x^2 + 4xM}{4x^2} \right)$$

Let us call this upper bound c^* .

Second, rho and gamma must be between 0 and 1.

$$\text{Rho } \rho = \frac{M}{Px}$$

$$0 < \rho < 1$$

$0 < \frac{M}{Px}$ is always satisfied.

$\frac{M}{Px} < 1$ implies:

$$M < Px$$

$$\text{Gamma } \gamma = \frac{1}{x(1-K)} K \left(-P\rho K + \frac{K}{B} P\rho(x - M) + \frac{K}{2} - x \right)$$

$\gamma > 0$ implies:

$$x^2 - x \left(\frac{K}{2} + \frac{KM}{B} \right) + \frac{KM^2}{B} + MK < 0$$

$$B \left(MK + x \left(x - \frac{K}{2} \right) \right) < KM(x - M)$$

If $MK + x \left(x - \frac{K}{2} \right) < 0$, this is always satisfied, so, if $M < \frac{x}{K} \left(\frac{K}{2} - x \right)$, gamma is greater than 0.

Let us call $< \frac{x}{K} \left(\frac{K}{2} - x \right) M_2$.

If $MK + x \left(x - \frac{K}{2} \right) > 0$, then:

$$B < \frac{KM(x - M)}{\left(MK + x \left(x - \frac{K}{2} \right) \right)}$$

Substituting B for its equilibrium expression:

$$\frac{1}{2x}(x - MK) - \frac{1}{2x}\sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)} < \frac{KM(x - M)}{\left(MK + x\left(x - \frac{K}{2}\right)\right)}$$

$$\frac{1}{2x}(x - MK) - \frac{KM(x - M)}{\left(MK + x\left(x - \frac{K}{2}\right)\right)} < \frac{1}{2x}\sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

$$(x - MK) - \frac{2xKM(x - M)}{\left(MK + x\left(x - \frac{K}{2}\right)\right)} < \sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

$$\frac{(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xKM(x - M)}{\left(MK + x\left(x - \frac{K}{2}\right)\right)} < \sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

If $(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xMK(x - M) < 0$, this is satisfied and gamma is greater than 0.

$$(x - MK)\left(x^2 - \frac{Kx}{2} + MK\right) < 2xKM(x - M)$$

$$x\left(x^2 - \frac{Kx}{2} + MK\right) - MK\left(x^2 - \frac{Kx}{2} + MK\right) < 2x^2KM - 2xKM^2$$

$$x\left(x^2 - \frac{Kx}{2}\right) + xMK - MK\left(x^2 - \frac{Kx}{2}\right) - M^2K^2 < 2x^2KM - 2xKM^2$$

$$0 < 2x^2KM - 2xKM^2 + M^2K^2 + MK\left(x^2 - \frac{Kx}{2}\right) - xMK - x\left(x^2 - \frac{Kx}{2}\right)$$

$$-M^2(2xK - K^2) + M\left(K\left(x^2 - \frac{Kx}{2}\right) - xK + 2x^2K\right) - x\left(x^2 - \frac{Kx}{2}\right) > 0$$

$$M^2(2xK - K^2) - M \left(K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K \right) + x \left(x^2 - \frac{Kx}{2} \right) < 0$$

If $2xK - K^2 > 0$

$$2xK > K^2$$

$$2x > K$$

$$x > \frac{K}{2}$$

This never holds, in this equilibrium, so this is a parabola turned upside-down. The solution is, therefore:

M

$$< \frac{K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K + \sqrt{\left(K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K \right)^2 - 4(2xK - K^2)x \left(x^2 - \frac{Kx}{2} \right)}}{2(2xK - K^2)}$$

OR

M

$$> \frac{K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K - \sqrt{\left(K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K \right)^2 - 4(2xK - K^2)x \left(x^2 - \frac{Kx}{2} \right)}}{2(2xK - K^2)}$$

$$K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K < 0$$

$$K \left(x^2 - \frac{Kx}{2} \right) < xK - 2x^2K$$

$$x^2 - \frac{Kx}{2} < x - 2x^2$$

$$-\frac{Kx}{2} < x - 3x^2$$

$$-\frac{K}{2} < 1 + 3x$$

$$-\frac{1}{3} - \frac{K}{6} < x$$

This always holds. So both zeros of the parabola are positive.

Let us call these M_3 and M_4 , respectively.

If $MK + x\left(x - \frac{K}{2}\right) > 0$ and $(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xMK(x - M) > 0$, gamma is positive if and only if:

$$c < \frac{(x - MK)^2 + 4xKM(x - M) - \left(\frac{(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xMK(x - M)}{MK + x\left(x - \frac{K}{2}\right)} \right)^2}{4x^2}$$

That is:

$$c < \frac{1}{4} - \frac{MK(2x - MK - 4x^2 + 4xM)}{4x^2} - \frac{\left(\frac{(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xMK(x - M)}{MK + x\left(x - \frac{K}{2}\right)} \right)^2}{4x^2}$$

Let us call this c^{***} .

To sum up, $\gamma > 0$ holds if:

- $M < M_2$ or
- $M > M_2$ but $M < M_3$ or

- $c < c^{***}$.

$$M_2 = \frac{x}{K} \left(\frac{K}{2} - x \right);$$

$$M_3 = \frac{K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K + \sqrt{\left(K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K \right)^2 - 4(2xK - K^2)x \left(x^2 - \frac{Kx}{2} \right)}}{2(2xK - K^2)},$$

$$M_4 = \frac{K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K - \sqrt{\left(K \left(x^2 - \frac{Kx}{2} \right) - xK + 2x^2K \right)^2 - 4(2xK - K^2)x \left(x^2 - \frac{Kx}{2} \right)}}{2(2xK - K^2)},$$

$\gamma < 1$ implies:

$$\frac{K^2 M(x - M)}{B} < x^2(1 - K) + x^2K - \frac{K^2 x}{2} + MK^2$$

$$\frac{K^2 M(x - M)}{B} < x^2 - \frac{K^2 x}{2} + MK^2$$

This implies that $\left(x^2 - \frac{K^2 x}{2} + MK^2 \right) > 0$, because the lhs is positive, by assumption. Rewriting, this implies that:

$$M > \frac{x}{2} - \frac{x^2}{K^2}$$

Let us call this M_1 .

This is only a necessary condition. Additionally, $\gamma < 1$ implies:

$$B \left(x^2 - \frac{K^2 x}{2} + MK^2 \right) > K^2 M(x - M)$$

$$B > \frac{K^2 M(x - M)}{\left(x^2 - \frac{K^2 x}{2} + MK^2 \right)}$$

Substituting B for its equilibrium expression:

$$\frac{1}{2x}(x - MK) - \frac{1}{2x}\sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)} > \frac{K^2M(x - M)}{\left(x^2 - \frac{K^2x}{2} + MK^2\right)}$$

$$\frac{1}{2x}(x - MK) - \frac{K^2M(x - M)}{\left(x^2 - \frac{K^2x}{2} + MK^2\right)} > \frac{1}{2x}\sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

$$(x - MK) - \frac{2xK^2M(x - M)}{\left(x^2 - \frac{K^2x}{2} + MK^2\right)} > \sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

$$\frac{(x - MK)\left(x^2 - \frac{K^2x}{2} + MK^2\right) - 2xK^2M(x - M)}{\left(x^2 - \frac{K^2x}{2} + MK^2\right)} > \sqrt{(x - MK)^2 + 4x(KM(x - M) - xc)}$$

This requires that $(x - MK)\left(x^2 - \frac{K^2x}{2} + MK^2\right) - 2xK^2M(x - M) > 0$, because the denominator of the lhs and the rhs are positive.

Then

$$(x - MK)\left(x^2 - \frac{K^2x}{2} + MK^2\right) > 2xK^2M(x - M)$$

$$x\left(x^2 - \frac{K^2x}{2}\right) + x(MK^2) - MK\left(x^2 - \frac{K^2x}{2}\right) - MK(MK^2) > 2xK^2M(x - M)$$

$$x\left(x^2 - \frac{K^2x}{2}\right) + MxK^2 - MK\left(x^2 - \frac{K^2x}{2}\right) - M^2K^3 > 2x^2K^2M - M^22xK^2$$

$$x\left(x^2 - \frac{K^2x}{2}\right) + MxK^2 - MK\left(x^2 - \frac{K^2x}{2}\right) + M^2(2xK^2 - K^3) > 2x^2K^2M$$

$$M^2(2xK^2 - K^3) + M\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) + x\left(x^2 - \frac{K^2x}{2}\right) > 0$$

Concavity of the parabola:

$$(2xK^2 - K^3) > 0$$

$$2xK^2 > K^3$$

$$2x > K$$

$$x > \frac{K}{2}$$

This does not hold in this equilibrium, so this parabola is turned upside-down.

Solution:

$$\frac{-\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) + \sqrt{\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right)^2 - 4(2xK^2 - K^3)x\left(x^2 - \frac{K^2x}{2}\right)}}{2(2xK^2 - K^3)}$$

< M

$$< \frac{-\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) - \sqrt{\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right)^2 - 4(2xK^2 - K^3)x\left(x^2 - \frac{K^2x}{2}\right)}}{2(2xK^2 - K^3)}$$

$$\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) > 0$$

$$\left(xK - \left(x^2 - \frac{K^2x}{2}\right) - 2x^2K\right) > 0$$

$$\left(K - \left(x - \frac{K^2}{2}\right) - 2xK\right) > 0$$

$$\left(K - x + \frac{K^2}{2} - 2xK\right) > 0$$

$$K + \frac{K^2}{2} - 2xK > x$$

$$K + \frac{K^2}{2} > x + 2xK$$

$$K + \frac{K^2}{2} > x(1 + 2K)$$

$$K \left(1 + \frac{K}{2}\right) > x(1 + 2K)$$

$$K \left(\frac{2 + K}{2}\right) > x(1 + 2K)$$

$$K \left(\frac{2 + K}{2(1 + 2K)}\right) > x$$

This always holds if $x < \frac{K}{2}$.

And both zeros are positive.

Let us call these M_5 and M_6 .

But this is also just a necessary condition. Additionally, $\gamma < 1$ imposes the following condition on c :

c

$$> \frac{((x - MK)^2 + 4xKM(x - M)) - \left(\frac{(x - MK) \left(MK^2 + x \left(x - \frac{K^2}{2} \right) \right) - 2xMK^2(x - M)}{MK^2 + x \left(x - \frac{K^2}{2} \right)} \right)^2}{4x^2}$$

That is:

$$c > \frac{1}{4} - MK \left(\frac{2x - MK - 4x^2 + 4xM}{4x^2} \right) - \frac{\left(\frac{(x - MK) \left(MK^2 + x \left(x - \frac{K^2}{2} \right) \right) - 2xMK^2(x - M)}{MK^2 + x \left(x - \frac{K^2}{2} \right)} \right)^2}{4x^2}$$

Let us call this c^{****} .

To sum up, $\gamma < 1$ requires:

- $M > M_1$ and
- $M_5 < M < M_6$ and
- $c > c^{****}$

$$M_1 = \frac{x}{2} - \frac{x^2}{K^2};$$

$$M_5 = \frac{-\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) + \sqrt{\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right)^2 - 4(2xK^2 - K^3)x\left(x^2 - \frac{K^2x}{2}\right)}}{2(2xK^2 - K^3)};$$

$$M_6 = \frac{-\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) - \sqrt{\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right)^2 - 4(2xK^2 - K^3)x\left(x^2 - \frac{K^2x}{2}\right)}}{2(2xK^2 - K^3)};$$

Q.E.D.

The relevant expressions for Lemma 5 are:

$$M_1 = \frac{x}{2} - \frac{x^2}{K^2};$$

$$M_2 = \frac{x}{2} - \frac{x^2}{K};$$

$$M_3 = \frac{K\left(x^2 - \frac{Kx}{2}\right) - xK + 2x^2K + \sqrt{\left(K\left(x^2 - \frac{Kx}{2}\right) - xK + 2x^2K\right)^2 - 4(2xK - K^2)x\left(x^2 - \frac{Kx}{2}\right)}}{2(2xK - K^2)};$$

$$M_5 = \frac{-\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right) + \sqrt{\left(xK^2 - K\left(x^2 - \frac{K^2x}{2}\right) - 2x^2K^2\right)^2 - 4(2xK^2 - K^3)x\left(x^2 - \frac{K^2x}{2}\right)}}{2(2xK^2 - K^3)};$$

$$c^{****} = \frac{1}{4} - MK \left(\frac{2x - MK - 4x^2 + 4xM}{4x^2} \right) - \frac{\left(\frac{(x - MK)\left(MK^2 + x\left(x - \frac{K^2}{2}\right)\right) - 2xMK^2(x - M)}{MK^2 + x\left(x - \frac{K^2}{2}\right)} \right)^2}{4x^2};$$

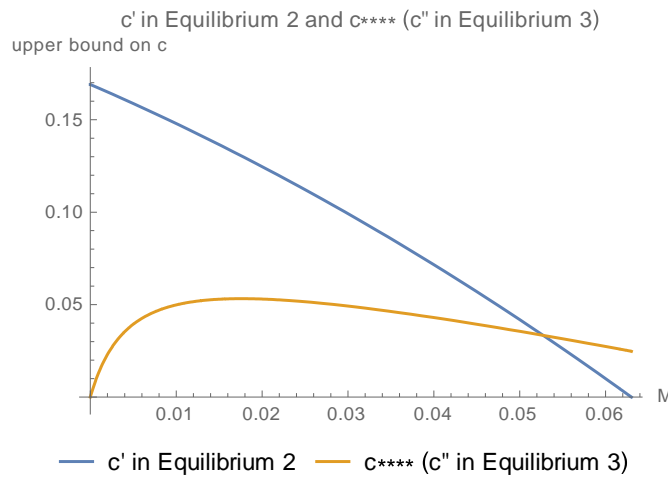
$$c^* = \frac{1}{4} - MK \left(\frac{2x - MK - 4x^2 + 4xM}{4x^2} \right);$$

$$c^{***} = \frac{1}{4} - MK \left(\frac{2x - MK - 4x^2 + 4xM}{4x^2} \right) - \frac{\left(\frac{(x - MK)\left(MK + x\left(x - \frac{K}{2}\right)\right) - 2xMK(x - M)}{MK + x\left(x - \frac{K}{2}\right)} \right)^2}{4x^2};$$

15. Proof of Lemma 6

Lemma 6: *There exists a set of parameters x , M , K and P for which c'' in Equilibrium 3 is higher than c' in Equilibrium 2.*

Example for $x = 0,09$; $K = 0,4$ and $P = 0,7$:



16. Proof of Lemma 7

(Figure 6: Inspection mechanism that implements lending for the highest c)

Lemma 7: *The inspection mechanism that restores lending for the highest c is:*

- Commitment, if $0 < x < \frac{K^2}{2}$
- Commitment, if $\frac{K^2}{2} < x < \frac{K}{4}$ and $M < \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}$
- Commitment, if $\frac{K}{4} < x < \frac{K}{2}$ and $M > \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}$
- No commitment, if $x > \frac{K}{2}$.

For the proof we need the following Lemmas:

Lemma 9: When no commitment is bounded above by c^* and commitment is bounded above by $c' = \frac{1}{4} - M$, commitment dominates if:

$$M < \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}, \text{ if } x < \frac{K}{4}$$

$$M > \frac{-4x^2\left(1+\frac{1}{K}\right)+2x}{K-4x}, \text{ if } x > \frac{K}{4}$$

Proof:

$$\frac{1}{4} - M > \frac{1}{4} - \frac{MK(2x - MK - 4x^2 + 4xM)}{4x^2}$$

$$\frac{MK(2x - MK - 4x^2 + 4xM)}{4x^2} > M$$

$$K(2x - MK - 4x^2 + 4xM) > 4x^2$$

$$K(-MK + 4xM) > 4x^2 - K(2x - 4x^2)$$

$$KM(-K + 4x) > 4x^2 - K(2x - 4x^2)$$

$$M(-K + 4x) > \frac{4x^2 - K(2x - 4x^2)}{K}$$

$$M(-K + 4x) > \frac{4x^2 + K4x^2 - 2xK}{K}$$

$$M(-K + 4x) > \frac{4x^2(1 + K) - 2xK}{K}$$

$$M(-K + 4x) > 4x^2 \left(\frac{1}{K} + 1 \right) - 2x$$

If $-K + 4x > 0$, that is, $x > \frac{K}{4}$, then:

$$M > \frac{4x^2 \left(\frac{1}{K} + 1 \right) - 2x}{(4x - K)}$$

If $x < \frac{K}{4}$, then:

$$M < \frac{4x^2 \left(\frac{1}{K} + 1 \right) - 2x}{(4x - K)}$$

Lemma 10: When no commitment is bounded above by c^* and commitment is bounded above

by $c' = \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4KM}{2x - K^2} \right)^2$, commitment dominates if:

$$M^2 \left(\frac{16x^2 K^2}{(2x - K^2)^2} + K^2 - 4xK \right) + M \left(4x^2 + \frac{8x^2 K}{2x - K^2} - 2xK + 4x^2 K \right) + x^2 < 0$$

The solution to this inequality is:

If $\left(\frac{16x^2 K^2}{(2x - K^2)^2} + K^2 - 4xK \right) > 0$:

M

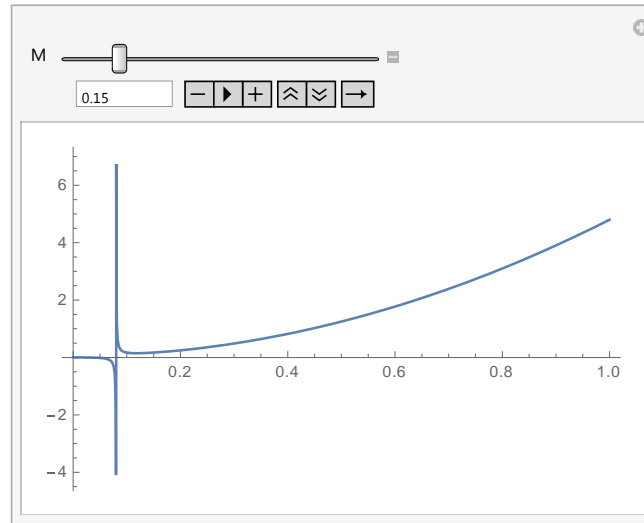
$$> \frac{-\left(4x^2 + \frac{8x^2K}{2x - K^2} - 2xK + 4x^2K\right) + \sqrt{\left(4x^2 + \frac{8x^2K}{2x - K^2} - 2xK + 4x^2K\right)^2 - 4\left(\frac{16x^2K^2}{(2x - K^2)^2} + K^2 - 4xK\right)x^2}}{2\left(\frac{16x^2K^2}{(2x - K^2)^2} + K^2 - 4xK\right)}$$

Or

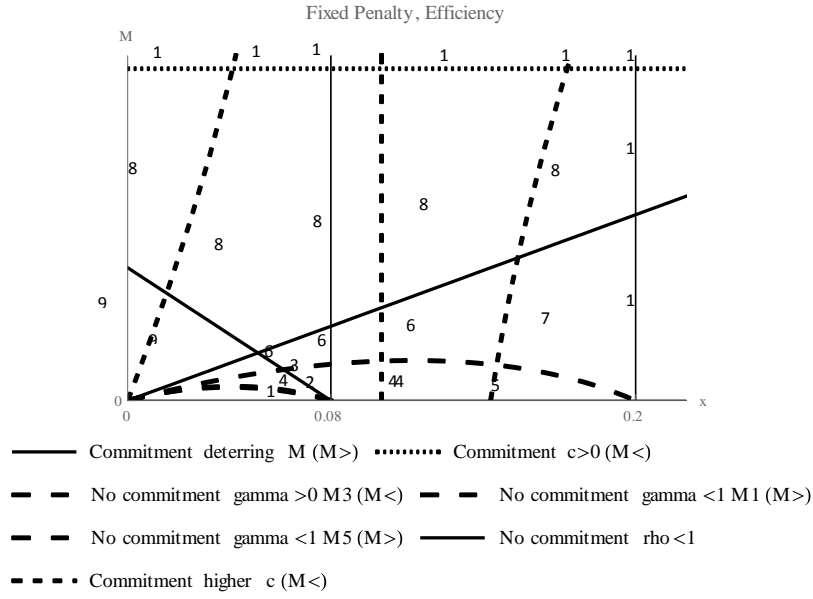
M

$$< \frac{-\left(4x^2 + \frac{8x^2K}{2x - K^2} - 2xK + 4x^2K\right) - \sqrt{\left(4x^2 + \frac{8x^2K}{2x - K^2} - 2xK + 4x^2K\right)^2 - 4\left(\frac{16x^2K^2}{(2x - K^2)^2} + K^2 - 4xK\right)x^2}}{2\left(\frac{16x^2K^2}{(2x - K^2)^2} + K^2 - 4xK\right)}$$

Still, using mathematica, we see that this expression is negative when $x < K^2/2$:



Using Lemmas 9 and 10, I prove **Lemma 7** (Figure 6):



In region 1, commitment dominates, because no commitment does not restore lending, there.

In region 2, commitment dominates by Lemma 4. In this region, commitment is bounded above

by $c' = \frac{1}{4} - M - \frac{1}{4} \left(1 + \frac{4KM}{2x - K^2} \right)^2$ and no commitment is bounded above c^* and $M^2 \left(\frac{16x^2 K^2}{(2x - K^2)^2} + K^2 - 4xK \right) + M \left(4x^2 + \frac{8x^2 K}{2x - K^2} - 2xK + 4x^2 K \right) + x^2 < 0$.

In region 3, commitment dominates, because dominance in region 2 implies dominance in region 3, since the upper bound on c under commitment is the same and the upper bound on c under no commitment is lower (under no commitment, c is now bounded by $c^{***} < c^*$).

In region 4, commitment dominates by Lemma 3. In this region, commitment is bounded by

$c' = \frac{1}{4} - M$ and no commitment is bounded by c^* and in this region $M < \frac{-4x^2 \left(1 + \frac{1}{K} \right) + 2x}{K - 4x}$, if $x < \frac{K}{4}$ and $M > \frac{-4x^2 \left(1 + \frac{1}{K} \right) + 2x}{K - 4x}$, if $x > \frac{K}{4}$.

In region 5, no commitment dominates by Lemma 3.

In region 6, commitment dominates, because dominance in region 4 implies dominance in region 6, since the upper bound under commitment is the same and the upper bound under no commitment is lower (under no commitment, it is $c^{***} < c^*$).

In region 7, commitment dominates, because dominance in region 6 implies dominance in region 7, since the upper bound on commitment and the upper bound on no commitment are unchanged.

In region 8, commitment dominates, because no commitment does not restore lending, there and commitment does.

In region 9, commitment dominates, because no commitment does not restore lending there and commitment does.

In region 10, no mechanism dominates the other, since both are a zero lending equilibrium, there.

In region 11, no commitment dominates, because the upper bound on c under no commitment ($\frac{1}{4}$) is higher than the upper bound under commitment ($\frac{1}{4} - M$).

Q.E.D.

Moreover, the requirement that the upper bound on c is positive is not binding in each relevant region:

First, $c = \frac{1}{4} - M$ is positive as long as $M < \frac{1}{4}$.

Second, as proved in proof 13 (Proof of Lemma 4), $c = \frac{1}{4} - M - \frac{1}{4}(1 + \frac{4MK}{2x-K^2})^2$ is positive in the parameter region where it is the binding upper bound on c .

Finally, $c^* = \frac{1}{4} - \frac{MK(2x - MK - 4x^2 + 4xM)}{4x^2}$ is positive, since the function qB at the prevailing market price ($q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$) has a positive maximizer and takes a positive value when $B = 0$ as long as $x > M$. Therefore, since qB is concave, its maximum is necessarily positive as long as $x > M$, which happens, since, in this equilibrium, $Px > M$.

Q.E.D.

17. Proof of Lemma 8

Lemma 8: When $P < \frac{1}{2}$, the regime that implements lending for the highest c for a type 1 government is “no mechanism” if $M > \max\{M', PK(\frac{2-PK}{4}), M_2, M_3\}$ or $\max\{M', M_2, M_3\} < M < \frac{K}{4} - \frac{x}{2K}$ when $x < \frac{K}{2}$ and if $M > \max\{M', PK(\frac{2-PK}{4}), x\}$ when $x > \frac{K}{2}$.

Lemma 8 results from the following Lemmas:

Lemma 11: When $P < \frac{1}{2}$, for given x , K , and M , a mechanism in which lenders commit to inspect the government restores lending for higher values of c than no mechanism if and only if $\frac{K}{4} - \frac{x}{2K} < M < PK(\frac{2-PK}{4})$.

Lemma 12: When $P < \frac{1}{2}$, for given x , K and M , a mechanism in which lenders do not commit to inspect restores lending for higher values of c than no mechanism if :

- $x < \frac{K}{2}$ and $M < M_3$ or
- $x > \max\{\frac{K}{2}, M\}$

Let us consider the case:

$$M < M_3$$

When the binding constraint in the no commitment mechanism is the price schedule, that is, for $M < M_3$, the commitment mechanism restores lending for higher values of c than no mechanism, because the price schedule is higher.

Price schedule under no mechanism:

$$q = 1 - B - PK$$

Price schedule under the no commitment mechanism:

$$q = 1 - B - P\rho K + \frac{K}{B}P\rho(x - M)$$

Since $\rho < 1$ in equilibrium and $x > M$ when $Px > M$, the price schedule under the no commitment mechanism is necessarily higher than that under no mechanism.

Lemma 13: Equilibrium 2 under the commitment mechanism provides lending for higher values of c than “no mechanism” if and only if:

Comparing binding upper bounds:

“No mechanism” Vs. commitment with hiding:

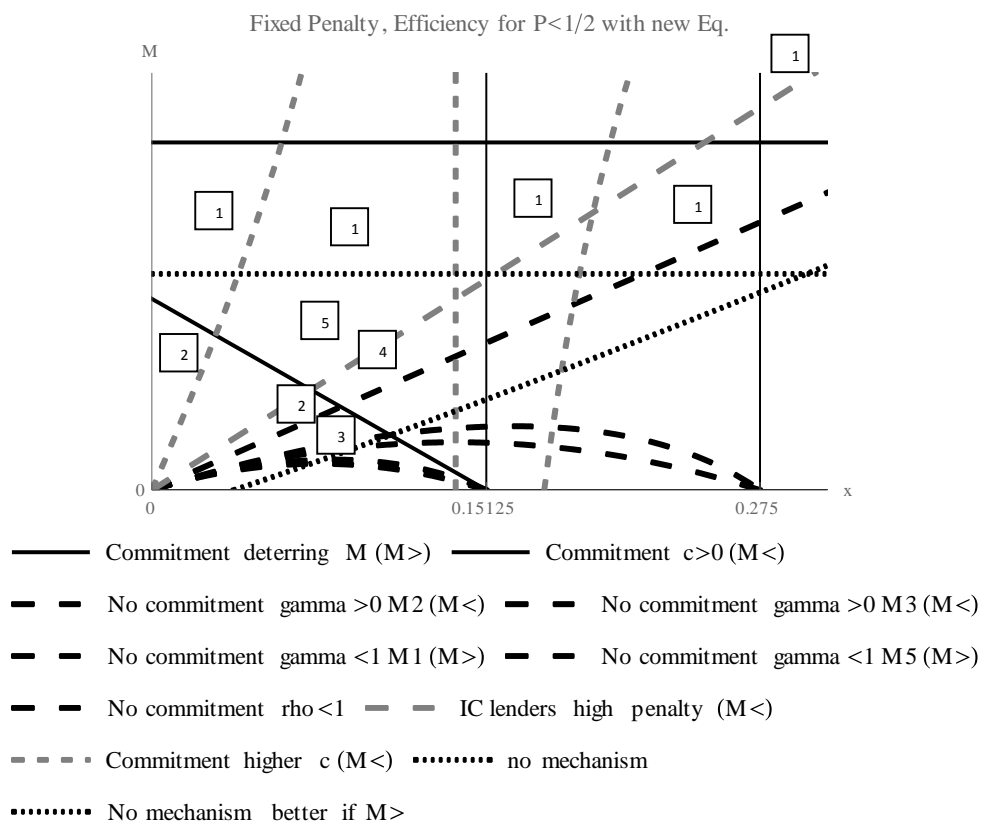
$$\begin{aligned} \frac{1}{4} - \frac{PK(2 - PK)}{4} &> \frac{1}{4} - M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px \\ - \frac{PK(2 - PK)}{4} &> -M - \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) + Px \\ \frac{1}{4} \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) &> \frac{PK(2 - PK)}{4} - M + Px \\ \left(1 - 2(1 - PK) \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right) + \left(\frac{4PKx - 4KM}{2x - K^2 + 2PK^2} \right)^2 \right) &> PK(2 - PK) - 4M + 4Px \\ 1 - 2(1 - PK) \left(\frac{4PKx}{2x - K^2 + 2PK^2} \right) + 2(1 - PK) \left(\frac{4KM}{2x - K^2 + 2PK^2} \right) + \frac{(4PKx - 4KM)^2}{(2x - K^2 + 2PK^2)^2} &> PK(2 - PK) - 4M + 4Px \end{aligned}$$

$$\begin{aligned}
& 1 - \frac{2(1-PK)4PKx}{2x-K^2+2PK^2} + \frac{2M(1-PK)4K}{2x-K^2+2PK^2} + \frac{(4PKx-4KM)^2}{(2x-K^2+2PK^2)^2} > PK(2-PK) - 4M + 4Px \\
& 1 - \frac{2(1-PK)4PKx}{2x-K^2+2PK^2} + \frac{2M(1-PK)4K}{2x-K^2+2PK^2} + \frac{16P^2K^2x^2 - 32PK^2xM + 16K^2M^2}{(2x-K^2+2PK^2)^2} > PK(2-PK) - 4M + 4Px \\
& 1 - \frac{2(1-PK)4PKx}{2x-K^2+2PK^2} + \frac{2M(1-PK)4K}{2x-K^2+2PK^2} + \frac{16P^2K^2x^2}{(2x-K^2+2PK^2)^2} + \frac{16K^2M^2 - 32PK^2xM}{(2x-K^2+2PK^2)^2} > PK(2-PK) - 4M + 4Px \\
& 1 - \frac{2(1-PK)4PKx}{2x-K^2+2PK^2} + \frac{2M(1-PK)4K}{2x-K^2+2PK^2} + \frac{16P^2K^2x^2}{(2x-K^2+2PK^2)^2} + \frac{16K^2M^2}{(2x-K^2+2PK^2)^2} - \frac{32PK^2xM}{(2x-K^2+2PK^2)^2} \\
& > PK(2-PK) - 4M + 4Px \\
& (2x-K^2+2PK^2)^2 - 2(1-PK)4PKx(2x-K^2+2PK^2) + 2M(1-PK)4K(2x-K^2+2PK^2) + 16P^2K^2x^2 \\
& + 16K^2M^2 - 32PK^2xM > (2x-K^2+2PK^2)^2(PK(2-PK) - 4M + 4Px) \\
& (2x-K^2+2PK^2)^2 - 2(1-PK)4PKx(2x-K^2+2PK^2) + 2M(1-PK)4K(2x-K^2+2PK^2) + 16P^2K^2x^2 \\
& + 16K^2M^2 - 32PK^2xM > (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px) - 4M(2x-K^2+2PK^2)^2 \\
& 16K^2M^2 - 32PK^2xM + 2M(1-PK)4K(2x-K^2+2PK^2) + 4M(2x-K^2+2PK^2)^2 + (2x-K^2+2PK^2)^2 \\
& - 2(1-PK)4PKx(2x-K^2+2PK^2) + 16P^2K^2x^2 > (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px) \\
& 16K^2M^2 + M(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2)) + (2x-K^2+2PK^2)^2 \\
& - 2(1-PK)4PKx(2x-K^2+2PK^2) + 16P^2K^2x^2 - (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px) > 0
\end{aligned}$$

This is a parabola turned upside, so this is satisfied if M is lower than the left-hand-side zero (which is negative) or higher than the right-hand-side zero, which I denote M' (its expression is written below (the lower expression, which is the right-hand-side zero)).

$$\begin{aligned}
M & < \frac{-(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2)) - \sqrt{(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2))^2 - 64K^2((2x-K^2+2PK^2)^2 - 2(1-PK)4PKx(2x-K^2+2PK^2) + 16P^2K^2x^2 - (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px))}}{32K^2} \\
M & > \frac{-(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2)) + \sqrt{(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2))^2 - 64K^2((2x-K^2+2PK^2)^2 - 2(1-PK)4PKx(2x-K^2+2PK^2) + 16P^2K^2x^2 - (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px))}}{32K^2} \\
M' & = \frac{-(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2)) - \sqrt{(4(2x-K^2+2PK^2)^2 - 32PK^2x + 2(1-PK)4K(2x-K^2+2PK^2))^2 - 64K^2((2x-K^2+2PK^2)^2 - 2(1-PK)4PKx(2x-K^2+2PK^2) + 16P^2K^2x^2 - (2x-K^2+2PK^2)^2(PK(2-PK) + 4Px))}}{32K^2}
\end{aligned}$$

So “no mechanism” implements lending for higher values of c if and only if $M > M'$.



Region 1: “no mechanism” achieves a higher upper bound and a higher price (since the payoff function of a type 1 is always the same, its payoff is determined by the price)

Region 2: “no mechanism” attains a higher upper bound on c and the price depends on c

Region 3: “no mechanism” attains a higher upper bound on c but a lower price

Region 4: “no mechanism” attains a lower upper bound on c and a lower price

Region 5: “no mechanism” attains a lower upper bound on c and a lower price